

OCCASIONAL PAPER NO. 5

A MARGINAL COST PRICING MODEL FOR GAS DISTRIBUTION  
UTILITIES

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## FOREWORD

The bylaws of The National Regulatory Research Institute state that among the purposes of the Institute is:

... to carry out research and related activities directed to the needs of state regulatory commissioners, to assist the state commissions with developing innovative solutions to state regulatory problems, and to address regulatory issues of national concern.

This report - the fifth in our series of Occasional Papers - helps meet that purpose, since the subject matter presented here is believed to be of timely interest to regulatory agencies and to others concerned with gas utility regulation.

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## PREFACE

In contrast to the case of electric utilities, there has been relatively little research in recent years on the application of marginal cost pricing principles to gas utilities, and most gas pricing studies have focused on the marginal cost of gas supply, discarding the marginal capacity cost as irrelevant because of an alleged excess capacity. However, several state regulatory agencies have recently expressed an interest in implementing marginal cost pricing for gas distribution utilities. For instance, the New York Public Service Commission issued, on September 17, 1979, Opinion No. 79-19 stating that the marginal cost of gas is a relevant consideration in gas rate cases, and requested estimates of the commodity and capacity marginal costs at different times, recognizing the effects of contract provisions with suppliers, of storage costs, and of plans for transmission, distribution and storage.

It is the purpose of this report to present a modeling methodology for the calculation of gas marginal costs at the distribution level, with particular emphasis on capacity costs. A partial equilibrium pricing model, including the optimization of supply mix and capacity expansion, the financial analysis of revenue requirements, and the design of marginal-cost-based rates that achieve the revenue requirement constraint, is developed and applied with data characterizing the East Ohio Gas Company. Average and marginal cost pricing policies are compared in terms of their respective impacts on total gas consumption, load factor, new plant investments, and consumers' surpluses. The marginal cost pricing policy is shown to significantly improve the utility's load factor, to require smaller investments in new plant, and to yield higher surpluses for both gas consumers and the utility.

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## I. Introduction

In contrast to the case of electric utilities, there has been relatively little research in recent years on the application of marginal cost pricing principles to gas utilities. While Tzoannos (1977) appears to be the only author accounting for gas capacity costs in a very simplified pricing model of the domestic gas system in Great Britain, most gas pricing studies in the U.S. (Blaydon et al., 1979; U.S. Department of Energy, 1980) focus on the marginal cost of gas supply, discarding the marginal capacity cost as irrelevant because of an alleged excess capacity. Nevertheless, it seems that comprehensive marginal cost pricing for gas distribution is gaining support in the U.S. For instance, the New York Public Service Commission issued, on September 17, 1979, Opinion No. 79-19 stating that the marginal cost of gas is a relevant consideration in gas rate cases, and requested explanations of calculations and estimates for the commodity and capacity marginal costs at different times, recognizing the effects of contract provisions with suppliers, of storage costs, and of plans for transmission, distribution, and storage. The commissioners also stated their awareness of the possibility that marginal-cost-based rates might provide excess revenues to the utilities, and of the need to deal with this issue, should it arise. It is also noteworthy that the effects of marginal cost pricing on the demand for natural gas and on changes in capital and operating utility costs constitute an important issue in the Public Utility Regulatory Policies Act of 1978 (Section 306--Gas Utility Rate Design Proposals).

It is the purpose of this paper to present a modeling methodology for the calculation of gas marginal costs at the distribution level, with

particular emphasis on capacity costs, and for the evaluation of the impacts of marginal-cost-based pricing policies in terms of energy conservation, utility plant requirements, and end-use economic efficiency. The remainder of the paper is organized as follows. The major conceptual and practical issues involved in applying marginal cost pricing principles to gas distribution utilities are analyzed in Section II. A literature review of existing gas systems planning and pricing models is presented in Section III. An overview of the proposed modeling methodology is presented in Section IV. The detailed structure of the model, as adapted to the East Ohio Gas Company (EOGC), is described in Section V, and the results of its application are presented in Section VI. Section VII concludes and outlines areas for further research.

## II. Conceptual and Practical Issues in Gas Distribution Marginal Cost Pricing

The various problems involved in the application of marginal cost pricing to gas distribution utilities can best be clarified when considering first the principles and results of a simple and general theoretical model of public utility pricing. Consider a utility supplying a commodity in amounts  $Q_1$  and  $Q_2$  during two distinct demand periods of equal duration,  $T_1$  (off-peak) and  $T_2$  (peak). These amounts are charged at prices  $P_1$  and  $P_2$ , and the demand functions  $P_1(Q_1)$  and  $P_2(Q_2)$  are assumed to be known. The utility operating costs are noted  $CO_1(Q_1)$  and  $CO_2(Q_2)$ . Under the assumption that no reserve margins are necessary, the utility's capacity must be equal to the peak demand  $Q_2$ , and the corresponding capacity cost is noted  $CC(Q_2)$ . The total welfare  $W$  for both the utility and its customers is equal to the sum of the corresponding producers' and consumers' surpluses, with:



$$W(Q_1, Q_2) = \int_0^{Q_2} P_1(Q) dQ + \int_0^{Q_2} P_2(Q) dQ - CO_1(Q_1) - CO_2(Q_2) - CC(Q_2) \quad (1)$$

The optimal production/consumption pattern is reached when  $W$  is maximized, i.e., when:

$$\frac{\partial W}{\partial Q_1} = P_1(Q_1) - \frac{dCO_1}{dQ_1} = 0 \quad (2)$$

$$\frac{\partial W}{\partial Q_2} = P_2(Q_2) - \frac{dCO_2}{dQ_2} - \frac{dCC}{dQ_2} = 0 \quad (3)$$

The derivatives of the operating and capacity costs are precisely the corresponding marginal costs, and are noted  $MCO_1(Q_1)$ ,  $MCO_2(Q_2)$ , and  $MCC(Q_2)$ . Equations (2) and (3) are restated as follows:

$$P_1(Q_1) = MCO_1(Q_1) \quad (4)$$

$$P_2(Q_2) = MCO_2(Q_2) + MCC(Q_2) \quad (5)$$

Equations (4) and (5), which highlight the close interrelationship between production, capacity investment and pricing, indicate that the optimal production/capacity pattern is obtained when (a) the off-peak price is equal to the off-peak marginal operating cost, and (b) the peak price is equal to the sum of the peak marginal operating cost and the marginal capacity cost.

Despite the simple and straightforward characteristics of the above framework, its application to gas distribution utilities entails several complications related to (a) the specification of gas demand, (b) the calculation of marginal costs for a given output pattern, (c) the determination of the optimal output/capacity/price pattern, and (d) the regulatory constraint on the utility's maximum revenue.

First, the demand for gas varies daily, weekly, and seasonally, depending mainly upon weather variability, with peak requirements during the winter season for space-heating purposes, and slack periods in the summer. The magnitude of this seasonal swing depends upon the characteristics and mix

of space-heating and other gas usages. The number of relevant demand periods is therefore larger than in the above example. In addition, this demand, even in a given period, is stochastic because of the randomness of temperature, hence the possible need to curtail this demand and to account for curtailment (or rationing) costs in establishing a pricing system. Finally, gas demand is highly spatialized as customers are distributed among the various communities (load centers) of the utility's service territory. These customers' requirements have spatially differentiated impacts on the cost of the distribution system. In short, both the temporal and spatial variability of gas demand must be accounted for in determining marginal costs of gas distribution.

Second, a gas distribution utility is a highly heterogeneous and complex production system which cannot be completely characterized by a few variables and cost functions, as hypothesized in the theoretical example. For a given demand/output pattern, the determination of the least-cost combination of production factors and of the corresponding marginal costs requires the consideration of all the subsystems making up the utility, such as its suppliers and its storage, transmission, and distribution plants, and of the cost trade-offs and interrelationships among them. A gas distribution utility generally receives most of its gas from one or more interstate pipelines, which generally apply a two-part rate system: a commodity rate, related to the amount of gas actually taken, and a demand rate, related to the contract demand defined as the maximum daily deliveries that the transmission pipeline commits itself to supply to the distributor. The demand rate provides for payment of the capacity (pipes, compressors, storage, etc.) that the pipeline has to install to honor the contract. In addition, many contracts also

involve a take-or-pay clause whereby the distributor commits itself to purchase a minimum quantity of gas or to pay for this minimum quantity if not actually taken. Other sources of gas supply may be local producers, natural gas produced by the company itself, and peak-shaving synthetic natural gas (SNG) plants owned by the company. The determination of the least-cost supply mix satisfying a given requirement pattern, which must account for the costs of and constraints bearing on the possible supply sources, is further complicated by the possibility, for the distributor, to develop and operate an underground storage system or to use, at a cost, the storage fields of other companies (very often its own suppliers). More gas than is needed by the end-use customers is purchased during the summer, and the excess gas is injected into storage at that time and withdrawn during the space-heating season, enabling the utility to contract for less peak demand, and hence to reduce the demand charges. Of course, storage is a beneficial operation only if storage costs are lesser than the reduction in demand charges, and the determination of the optimal trade-off is subject to several supply and storage technological constraints. The trade-off analysis must further account for the location of the supply take-off points, where gas is physically received from the suppliers, for the location of the load centers where gas is injected into the local distribution networks, for the location of the storage fields, and for the design of the network of transmission lines that convey gas at high pressure between these various nodes. The transmission lines may be equipped with compressors, and the well-known trade-off between pipe diameter, gas flow, pressure drop, and compression ratio and power must be included in the analysis. In summary, the utility planner is facing a large number of decision variables in designing the system that will satisfy, at least

cost, gas requirements specified both geographically and temporally. This set of decision variables may vary significantly among utilities, depending in particular upon whether the future system may be designed without any constraint or whether the system's expansion is severely constrained by the characteristics of the existing system (e.g., existing transmission lines and storage pools, non-renegotiable purchases agreements, etc.). The decision variables may include the amounts of gas to be purchased from each supplier at each take-off point during each period, the maximum daily deliverability of each supplier, the location and diameter of the pipe links making up the transmission network, the location and power of the compressors, the location and capacity of the storage fields, the amount of gas conveyed in each transmission link during each period, the periodic storage injections and withdrawals, etc. Several constraints must be accounted for, such as minimum and maximum pressures in the pipes and storage reservoirs, maximum available supplies, maximum pipe and compressor capacities, maximum storage deliverability, flow balances at the different nodes of the network, etc. Obviously, the optimal design cannot be determined intuitively and must be the output of a mathematical programming model minimizing the total system cost subject to several constraints. This model may be solved exactly or only sub-optimally through some heuristic procedure, depending upon its structure and the simplifications made. If the model turns out to be a linear program, then the shadow prices of the spatially and temporally defined requirement constraints are exactly equal, at the optimum, to the marginal costs associated to marginal variations of these requirements. Such an approach to the calculation of space-time marginal costs has been applied by Scherer (1976)

to the case of electricity generation, transmission, and pumped storage. However, when the system cannot be reduced to a linear format, a possible approach is to solve the model while increasing, alternatively, each requirement by an increment  $\Delta D$ . The resulting cost increment  $\Delta C$  leads to an approximation of the corresponding marginal cost, with  $MC \approx \Delta C / \Delta D$ . Obviously, the above marginal costs would encompass supply, storage, and transmission marginal costs. However, providing for the increments  $\Delta D$  implies also additional distribution capacity costs within the load centers. Conceptually, then, the internal structure of each load center should also be formalized as a network serving all the individual customers (residential, commercial, industrial), and the marginal distribution cost corresponding to the marginal variation of the demand of any customer should be computed through a procedure similar to the one discussed for the larger network. Through such a hierarchical analysis, the total marginal cost corresponding to any marginal variation in demand could be calculated. Whether such a comprehensive model is practically feasible or whether simplifying assumptions are necessary will be determined, in part, through the review of the literature on gas utility models presented in Section III.

Third, in order to determine the optimal welfare solution it is necessary to interface the space-time demand functions with the marginal costs calculation procedure outlined previously, and to devise an iterative scheme until the quantities demanded are exactly equal to the optimal levels of outputs. Such a scheme is conceptually equivalent to solving equations (3) and (4). However, it is possible that no convergence is obtained because of peak shifting. Indeed, it may happen that consumers,

reacting to the new peak and off-peak prices, shift their demands in such a way that the former off-peak period becomes the new peak one. In such a case the original prices would no longer be equal to the marginal costs corresponding to the new demand pattern.

Finally, it is necessary to make sure that the utility's revenues generated through marginal cost pricing do not exceed the maximum allowed revenues as determined through traditional rate base regulation. This revenue constraint may require an adjustment of the pricing system, and the implications of this adjustment must also be analyzed.

### III. Review of the Literature

In order to assess the prospects for developing an operational model of gas distribution marginal cost pricing that accounts for the factors analyzed in the previous section, it is first necessary to review the literature on gas systems models. These models can be classified according to several criteria. A first criterion is whether the model characterizes the whole industry or the individual company. In the latter case, a second criterion is whether the model focuses on engineering system design, resource allocation, shortage management, or pricing. A third criterion is the extent of spatial and temporal disaggregation of the model.

Industry-wide market simulation models of gas supply and demand have been developed by MacAvoy and Pindyck (1973) and by Murphy et al. (1981), among others. The latter incorporated a gas submodel in the Project Independence Evaluation System (PIES). The purpose of both modeling efforts was to assess the impacts of Federal policies related

to gas pricing at the wellhead (deregulation) and at the transmission-distribution levels (incremental versus average cost pricing). In these studies, individual companies are aggregated regionally, and are, at best, represented by regional markup equations, with no cost analyses or modeling at the firm level. The PIES modeling approach uses a linear program to optimize energy supplies and identifies the relevant dual variables as price inputs to econometrically estimated demand functions. An iterative procedure is applied until market equilibrium is reached.

Optimization models dealing with the design of transmission pipelines make up for a significant share of the literature on gas systems planning models. These models generally focus on selecting the locations and diameters of pipeline segments, and the numbers, locations, and capacities of compressor stations, that minimize capital and operating costs subject to flow and supply/delivery constraints. They use such techniques as dynamic programming (Wong and Larson, 1968), non-linear programming (Flanigan, 1972, Edgar et al., 1978), or heuristic procedures (Rothfarb et al., 1970). A multi-period extension of such pipeline models, including the simultaneous determination of optimal production rates for supply reservoirs, and optimal flows for storage reservoirs, has been developed by Heideman (1972), using both linear and non-linear programming.

At the distribution level, gas systems planning models may be classified as (1) short-term operating policy models, (2) long-term operating and investment policy models, and (3) shortage management models. Slater et al. (1978) have developed a spatialized and very detailed model of a distribution utility, based on daily simulation of individual storage fields, compressors, regulator valves, and pipeline links. This model provides for pressure calculations node by node, and produces a gas balance

sheet typical of those published daily by gas distributors. It is to help the gas dispatcher in testing alternative flow routing policies, but is inappropriate for dealing with longer-term decisions. All the longer-term policy models that were reviewed feature the company in an aggregate, non-spatialized fashion. Levary and Dean (1980) developed a deterministic linear program to optimize storage and purchases decisions, either minimizing costs or minimizing shortages. Storage investment decisions and optimal supply contracts selection are incorporated in the chance-constrained programming model developed by Guldman (1983). This model accounts explicitly for service reliability effects related to weather randomness. Long-term market expansion policies are evaluated in terms of financial, adequacy of service, and economic efficiency criteria by Guldman and Czamanski (1980) with a simulation model based on economic, engineering, accounting, and regulatory relationships. Although this model was not developed to test alternative pricing policies, it includes an average cost pricing module linked to market share and gas demand equations. Finally, gas shortage management models have been developed by O'Neill *et al.* (1979) at the regional, multi-firm level, and by Guldman (1981a) at the utility level. These models determine the optimal allocation of the available gas when a deficit between supply and demand develops.

Besides the Guldman/Czamanski's (1980) model, all the above models do not involve any pricing considerations. Gas requirements are given exogenously, and the problem is to optimize some criterion subject to the satisfaction of these requirements. Tzoannos (1977) is apparently the only author to account simultaneously for pricing and production/investment decisions in a very simple model of the domestic gas market in Great Britain.



His problem is to determine the four seasonal production/consumption levels and the seasonal production capacity that maximize a welfare function similar to equation (1) subject to four seasonal capacity constraints. Linear gas demand functions are estimated for each quarter and used in conjunction with linear energy and capacity cost functions, leading to the formulation of a quadratic program. The results indicate a substantial improvement in capacity utilization and a net gain in welfare (with some transfer of surplus from producers to consumers) under the optimal (i.e., peak-load) pricing policy as compared to the actual policy. The major shortcomings of this model are (1) the assumption of a homogeneous production system, (2) the very high level of temporal and spatial aggregation, and (3) the absence of seasonal storage options. The model designed by ICF, Incorporated, for the U.S. Department of Energy (1980) appears to be the only other endeavor to empirically estimate gas marginal costs at the distribution level. It was developed within the framework of the Natural Gas Rate Design Study conducted by the U.S. Department of Energy under mandate of Section 306 of the Public Utility Regulatory Policies Act of 1978. An overview of the approach can also be found in Blaydon et al. (1979). This is a year-by-year simulation model designed to find equilibrium points in supply and demand and to assess quantitatively the impacts of alternative rate structure. It involves an energy supply cost minimization submodel which yields, as a by-product, the marginal costs of supply for each of the five segments of the load duration curve. A pricing policy based on these marginal costs has been considered. However, marginal capacity costs have been discarded because of alleged excess capacity, and so were the other operating marginal costs under the assumption that such costs are fixed over a large range of supply volumes. No plant expansion

is considered besides adding (1) new customer plant, taken as proportional to the number of new customers, and probably including such items as services, meters, and local mains, and (2) replacement plant taken as a fraction of the depreciated plant. Storage capacity development is not considered as an option, and seasonal storage space is used at a fee. In addition, the load duration curve approach sorts loads independently of their chronological occurrence, thus distorting the timing of demand and, in turn, adversely affecting storage, supply and allocation decisions. Nevertheless, the ICF model, despite the above shortcomings, constitutes a contribution to the field, in particular with respect to its treatment of gas market sharing, cost allocation to rate classes, and rate design.

#### IV. Overview of the Gas Marginal Cost Pricing Model

The previous review suggests the following components of a comprehensive analysis of gas distribution marginal cost pricing: (1) a gas system optimization analysis of all the relevant trade-offs between supply mix and production, storage, transmission, and distribution plants capacity expansion, accounting for both the temporal and spatial dimensions, and yielding the marginal costs of any given pattern of gas demand; and (2) a market equilibrium analysis, where demand and supply would be interfaced, and demand would depend upon prices based upon marginal costs.

What are the practical prospects for developing a complete gas distribution system optimization model? While the available literature suggests some approaches to the simultaneous optimization of supply, storage, and transmission, no model could be found that optimizes the design and operation

of a distribution network in an urban area (i.e., a load center). It is thus unlikely that the distribution plant could be optimized simultaneously with the other components of the system. In addition, the review of the optimization models indicates that the solution of a model accounting for all the possible decision variables is, given the state of the art in mathematical programming, close to impossible, due to the highly combinatorial and non-linear character of the system, and that suboptimal heuristic solution procedures would be necessary.

In view of the above-mentioned problems, a simplified, aggregated and non-spatialized optimization submodel has been developed to calculate the marginal supply, storage, and transmission costs. This submodel is cast into a linear programming format and yields monthly marginal costs, that are complemented by the marginal costs of the other, non-optimized system components within the framework of an integrating market equilibrium simulation model. The approach can be characterized as static, as the analysis applies to a horizon year for which all the relevant forecasts are assumed available. A general flow diagram of the model is presented in Figure 1. It consists of three major, interlinked blocks: (1) Exogenous Data and Assumptions (EDA), (2) Average Cost Pricing Policy (ACPP), and (3) Marginal Cost Pricing Policy (MCP).

The EDA block includes: (1) market-related parameters such as sectoral market growth rates, base and space-heating load coefficients, and price elasticities of monthly gas demands; (2) supply-related parameters such as maximum supplies and rates for the different possible suppliers; and (3) utility-related parameters such as operating and capacity unit costs, maximum capacity expansions, the allowed rate of return, and other financial parameters (tax rates, etc.).

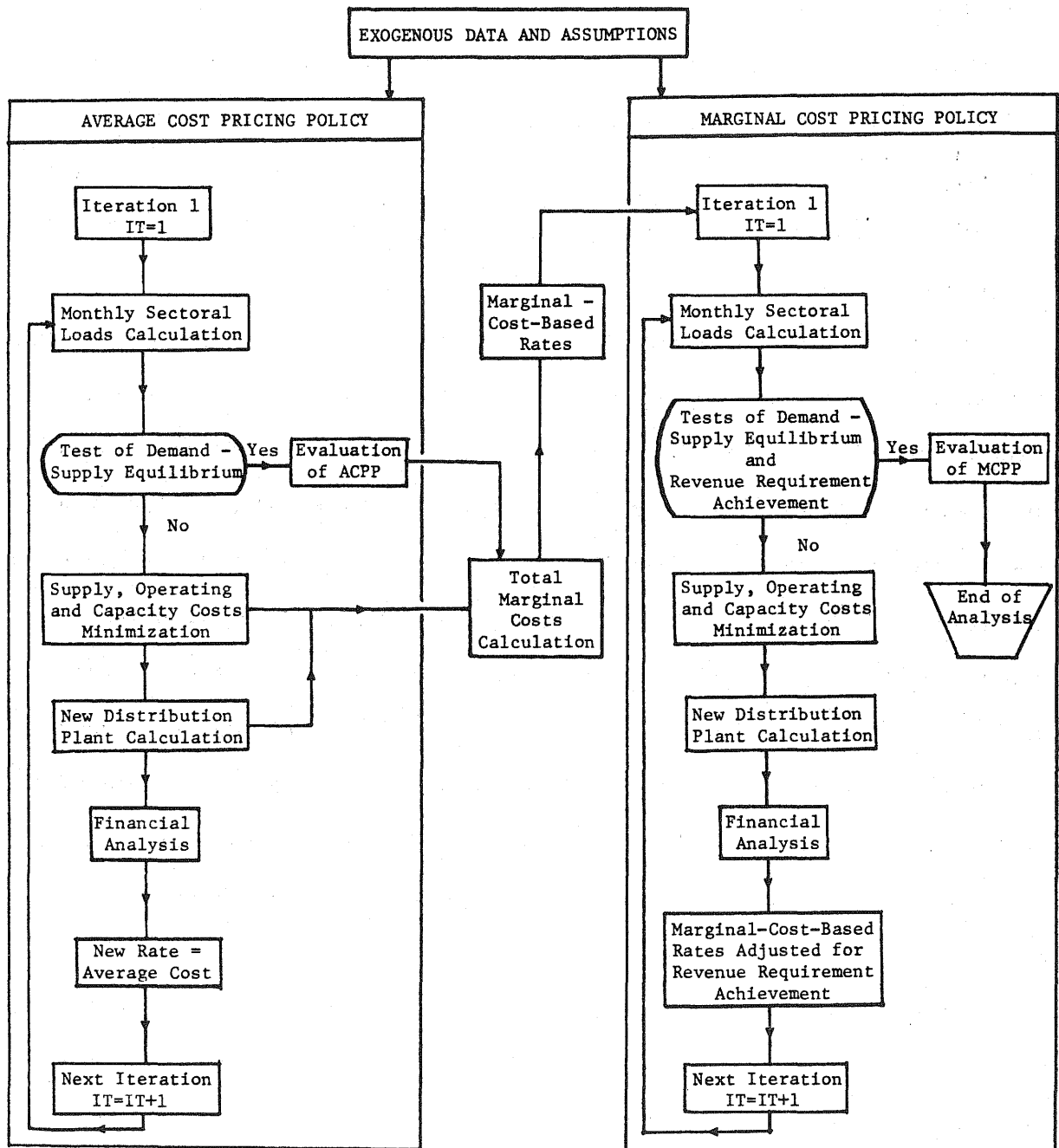


Figure 1 Structure of the Gas Marginal Cost Pricing Model

The above data and assumptions are first used in the ACPP block, where the monthly loads of the residential, commercial, and industrial sectors are calculated while using an initial exogenous value of the gas rate applied uniformly to all sectors and in all months. These loads are then inputs to the utility supply, operating and capacity costs minimization submodel, which determines the optimal trade-off between supply mix and own-production, storage and transmission operations and capacity expansion decisions, subject to satisfying the above-mentioned loads and various utility-related technological constraints, and which yields shadow prices for the monthly load constraints. These marginal costs are complemented by other marginal costs such as the distribution marginal costs computed in the next step, together with the total new distribution plant. The total new plant (production, storage, transmission, distribution) is then calculated in the financial analysis submodel, which closely replicates the computations typically made in the context of rate cases. The utility's rate base is first calculated, and then so is the revenue from gas sales necessary to provide the allowed rate of return on this rate base. This revenue, divided by the total annual gas load, yields the necessary average volumetric rate. This rate is used as the new rate for the calculation of the monthly sectoral loads in the next iteration. This iterative procedure ends when the difference between the demands of two consecutive iterations does not exceed an exogenously prescribed small value. Note that, by virtue of the method of computing the average rate, the revenue requirement objective is necessarily achieved at the end. The equilibrium average cost pricing policy is then evaluated with respect to several criteria, such as (1) total annual gas requirements, (2) peak monthly load, (3) load factor, (4) new plant investments, and (5) sectoral and total consumers surpluses. This evaluation is to provide benchmarks for the assessment of marginal cost pricing policies.

The total monthly marginal costs corresponding to the ACPP equilibrium are then computed and used to design monthly rates either equal to these marginal costs or based on them, according to adjustment procedures discussed later on. These rates are then inputs to the first iteration of the MCPP block, which consists in the repetition of a calculation cycle similar to that of the ACPP block, the major difference being that rates are now based on marginal costs and are no longer equated to the average cost. Therefore, the revenue requirement constraint is very unlikely to be achieved, and an additional rate adjustment mechanism is considered, based on the difference between the revenue requirement goal and the actual revenue. New rates are computed at the end of each cycle and are used to compute the monthly sectoral loads at the beginning of the next cycle. If the new loads are equal to the loads computed in the previous iteration and if the revenue requirement objective is achieved, the iterative procedure is terminated, and the final pricing, output and investment pattern is evaluated with respect to the same criteria as used in the ACPP analysis.

There are significant variations in the structure of gas distribution utilities in terms of their supply mix (number of suppliers, maximum supplies, rate structure, take-or-pay clauses, etc.), their own gas production and storage system (or the storage space they are able to rent), and the extension of their transmission system. It is therefore difficult to characterize such diverse companies by a set of prototypical or synthetic utilities, and it is thus necessary to adapt the above-outlined modeling methodology, and in particular its cost minimization submodel, to the specific features of the utility considered. The remainder of this paper describes the application of the model to the East Ohio Gas Company (EOGC), which serves the northeastern

part of Ohio, including the cities of Cleveland, Akron, Canton, Warren, and Youngstown. It is one of the largest gas distribution utilities in Ohio, with 908,758 residential customers, 52,867 commercial customers, and 1,108 industrial customers in 1977, the base year for which most of the data have been prepared. The raw data have been drawn from the Annual Reports (1970-1977) of the EOGC to the Public Utilities Commission of Ohio (PUCO) or have been obtained directly from the company's management.

The EOGC is a complex and rather "complete" utility, in that it has nearly all the functions a gas distribution utility can display, in particular a diversified supply mix, and natural gas production, storage, and transmission systems. Hence taking the EOGC model as benchmark and starting point, the application of the methodology to a simpler utility would involve (1) the scaling down of the EOGC model by deleting its components irrelevant to the simpler utility, and (2) the preparation of new input data.

## V. Structure of the Gas Marginal Cost Pricing Model

### 5.1. The Monthly Load Submodel

Gas end-users are customarily grouped into three sectors--residential, commercial, and industrial--and monthly gas demand (load) functions are developed for each sector, accounting for market size, weather pattern, and gas prices. The general formulation of the load function for month  $m$ ,  $DG_m$ , is assumed to be:

$$DG_m = N * F(DD_m) * G(\bar{P}) \quad (6)$$

$$\bar{P} = (P_1, \dots, P_m, \dots, P_{12})$$

where  $DD_m$  is the number of heating degree-days during month  $m$ ,  $P_m$  the price charged for gas during that month, and  $N$  the number of sectoral customers.

Such a formulation is consistent with the results of several energy demand (Nelson, 1975) and gas demand (Berndt and Watkins, 1977; Neri, 1980) econometric analyses. The specification of these load functions for the EOGC is the outcome of a synthesis based on (1) a review of previous research on gas demand modeling, and (2) EOGC load data analyses.

There is very little research available on the relationship between gas demand and price at the intra-annual (i.e., seasonal, monthly, etc.) level, and the bulk of existing studies focuses on the determinants of total annual demand, both in the short and long terms, with the exception of Neri (1980) who developed seasonal demand functions for the residential sector, using a cross-section of 1108 households and applying a log-linear specification. His results imply a unit elasticity for heating degree-days in winter, suggesting that the weather component in Equation (6) is linear in degree-days. As could be expected, Neri found the degree-day variable insignificant in summer. Testing alternative sets of explanatory variables, Neri obtained short-term elasticity estimates with respect to the marginal price of gas ranging from  $-.18$  to  $-.30$  during the winter season, and from  $-.18$  to  $-.23$  during the summer season. The wide ranges of elasticity estimates obtained with annual demand analyses are underscored in the comprehensive review of 25 different studies presented in the final report (Appendix C - pp. 68) of the Natural Gas Rate Design Study (U.S. Department of Energy, 1980). The ranges and mean values of these elasticity estimates are reported in Table 1.



Table 1 Summary of Short-Run and Long-Run Elasticity Estimates

Sector	Short Run		Long Run	
	Range	Mean	Range	Mean
Residential	0 to -0.633	-0.240	0 to -2.20	-0.88
Commercial	-0.274 to -0.380	-0.317	-0.741 to -1.45	-1.12
Industrial	-0.070 to -0.170	-0.116	-0.44 to -1.98	-1.17

Source: Natural Gas Rate Design Study - U.S. Department of Energy (1980)

What should then be the specification of the price component  $G(\bar{P})$  in Equation (6)? A first issue is whether long-term or short-term adjustments in demand should be considered. Although the present study refers to a long-term planning horizon, long-term adjustments in gas demand in response to price changes (*i.e.*, adjustments in the stock of gas appliances, energy conservation investments, etc.) are, to a large extent, irrelevant to the purposes of the study. Indeed, long-term adjustments are mainly induced by the average level of gas prices and its comparison with the prices of alternative energy sources and the costs of conservation measures. Because of the revenue constraint included in the model, the equivalent average price under any marginal cost pricing policy will be close to the uniform rate implemented under the average cost pricing policy. Hence, the long-term market adjustments are likely to be similar under both pricing approaches, and can be viewed as captured by the market size parameter  $N$  in Equation (6).<sup>1</sup> While it is clear that only short-term adjustments in demand should be considered for the residential and commercial sectors, the short-term elasticities indicated

<sup>1</sup> It is quite possible that some long-term adjustments may be specifically induced by a time-differentiated pricing policy, particularly if it involves large price differentials. Unfortunately, empirical studies on this subject do not exist to the best of our knowledge, and their future availability will depend upon observing market behavior under such new pricing policies.

in Table 1 for the industrial sector probably underestimate short-term, temporary fuel switching possibilities in many industrial activities where boilers may easily be equipped with different types of burners. Such multi-fuel burning capabilities have been developed by many industries to reduce the impact of temporary or chronic gas curtailments.

It is assumed that demands are independent across periods, which is probably realistic with monthly periods, but would no longer be so with much shorter ones (e.g., an hour), and that the demand functions are of the constant-price-elasticity form, which is consistent with the results of most previous studies. The elasticities of the commercial and industrial sectors are assumed to be the same throughout the year. A value of -0.32 is selected for the commercial sector, close to the mean value indicated in Table 1. In order to account for short-term industrial fuel substitution, the mean of the short-run and long-run average industrial elasticities indicated in Table 1 has been selected, with a value of -0.64. In the case of the residential sector, the elasticities were taken equal to -0.20 during the summer season (May through October) and to -0.24 during the winter season (November through April). These values have been selected as the mid-points of the seasonal elasticity intervals delineated by Neri (1980). It is, however, clear that there is much uncertainty about all these elasticity estimates, calling for additional empirical research as well as sensitivity analyses.

The weather-related component of Equation 6,  $F(DD_m)$ , was obtained by regressing the observed 1972 sectoral monthly loads on the corresponding monthly numbers of heating degree-days. The year 1972 was selected because it was the most recent one (as from 1977, the base year of the analysis) without significant curtailments of the industrial customers, whose actual

usage then closely approximated their potential demand. The resulting regression equations, with  $R^2$  coefficients equal to 0.989 for the residential and commercial sectors, and to 0.920 for the industrial sector, were adjusted for the change in the numbers of customers from 1972 to 1977, with:

$$DGR_m^{\circ} = 908,758 * (3.5895 + 0.02679 * DD_m) \text{ (MCF)} \quad (7)$$

$$DGC_m^{\circ} = 52,867 * (29.2937 + 0.17584 * DD_m) \text{ (MCF)} \quad (8)$$

$$DGI_m^{\circ} = 1,108 * (8357.3596 + 2.92857 * DD_m) \text{ (MCF)} \quad (9)$$

where (a)  $DGR_m^{\circ}$ ,  $DGC_m^{\circ}$ , and  $DGI_m^{\circ}$  are the residential, commercial, and industrial loads during month  $m$  of the base year (1977), (b) the first component of each equation is the base year number of customers, and (c) the second component of each equation is the monthly load per customer expressed as a linear function of the monthly number of degree-days, with the first coefficient representing the base load, independent of weather, and the second one the space-heating load per customer. For an average annual number of 6258 degree-days, the residential, commercial, and industrial base loads correspond to 20.5%, 24.2%, and 84.5% of the total sectoral loads, respectively. The values of  $DGR_m^{\circ}$ ,  $DGC_m^{\circ}$ , and  $DGI_m^{\circ}$  are estimated at the 30-year average values of the monthly degree-days  $DD_m$ , as presented in Table 2. Monthly demands are therefore treated as deterministic variables.

Actually, weather randomness is reflected in the stochastic character of the variables  $DD_m$ , which are independent and normally distributed (Guldmann, 1981). While the integration of stochastic demands and reliability considerations into the present methodology would clearly be desirable, such an endeavor calls for additional research. As a first step, the use of an average demand pattern should provide the general gas pricing policy assessment aimed at in the present study.

Table 2 Average Monthly Numbers of Heating Degree-Days

Month	Degree-Days	Month	Degree-Days	Month	Degree-Days
January	1207.7	May	248.2	September	120.5
February	1046.3	June	50.5	October	371.6
March	892.5	July	11.0	November	712.6
April	506.6	August	18.9	December	1071.6

In order to formulate the monthly sectoral load functions  $DGR_m$ ,  $DGC_m$ , and  $DGI_m$  for the planning year, it is necessary to integrate market growth, weather, and price effects, with:

$$DGR_m = (1 + RMR) * DGR_m^o * \left(\frac{P_m}{P_A}\right) - \begin{cases} 0.20 & (\text{May} \rightarrow \text{October}) \\ 0.24 & (\text{November} \rightarrow \text{April}) \end{cases} \quad (\text{MCF}) \quad (10)$$

$$DGC_m = (1 + RMC) * DGC_m^o * \left(\frac{P_m}{P_A}\right)^{-0.32} \quad (\text{MCF}) \quad (11)$$

$$DGI_m = (1 + RMI) * DGI_m^o * \left(\frac{P_m}{P_A}\right)^{-0.64} \quad (\text{MCF}) \quad (12)$$

where RMR, RMC, and RMI are the residential, commercial, and industrial sector growth rates between the base year and the planning year, and  $P_A$  a scaling factor taken equal to the average cost entailed by the demand pattern  $\{DGR_m^o, DGC_m^o, DGI_m^o, m = 1 \rightarrow 12\}$ .

## 5.2. The Supply, Operating, and Capacity Costs Minimization Submodel

### 5.2.1. Overview of the submodel

The decision variables of the costs minimization submodel include (1) the pipelines, well-head, and field-line monthly purchases, (2) the maximum deliveries contracted with the pipelines, (3) the capacity expansion of the natural gas production plant and the monthly levels of gas produced, (4) the capacity expansion of the storage plant and the monthly storage deliveries

and withdrawals, and (5) the capacity expansion of the transmission plant. The submodel, formulated as a linear program, minimizes the sum of purchases, production investment and operation, storage investment and operation, and transmission investment costs, subject to several constraints related to (1) maximum monthly and annual purchases, (2) maximum production and storage capacity expansions, (3) maximum monthly storage deliveries and withdrawals, (4) monthly and annual production rates, (5) maximum monthly transmission flows, and (6) the satisfaction of the monthly gas requirements of the end-use customers.

### 5.2.2 Gas supply modeling

Historically, the EOGC has purchased, on the average, about 90% of its annual supply from two interstate pipeline companies: Consolidated Gas Supply Corporation (75%) and Panhandle Eastern Pipeline Company (15%). The remainder was obtained from well-head and field-line purchases from local Ohio producers. These four sources of supply are the only ones considered in the present model.

The monthly purchases from Consolidated and Panhandle are noted  $SUP1_m$  and  $SUP2_m$  for month  $m$ , respectively. In order to keep up with seasonal definitions and constraints, the year is defined as the period spreading from April 1 to March 31 (with months numbered accordingly). It is assumed that there are limits,  $SUP1T$  and  $SUP2T$ , to the total annual supplies purchasable from Consolidated and Panhandle, respectively. Hence the constraints:

$$\sum_{m=1}^{12} SUP1_m \leq SUP1T \quad (13)$$

$$\sum_{m=1}^{12} SUP2_m \leq SUP2T \quad (14)$$

The rate structure of Consolidated includes a commodity charge, CC1, related to the amount actually purchased, a demand charge, DC1, related to the maximum contracted daily purchase DAYMX1, and a winter requirement charge, WRC, related to total winter gas purchases (from November 1 to March 31). The rate structure of Panhandle, in addition to a commodity charge, CC2, and a demand charge, DC2, includes a take-or-pay clause stating that the minimum monthly bill must include a minimum commodity charge based upon 75% use of the demand contract DAYMX2. The demand contracts DAYMX1 and DAYMX2 are decision variables. Assuming that the monthly purchases  $SUP1_m$  and  $SUP2_m$  are uniformly spread over the month, the following maximum monthly purchase constraints must hold for each month  $m$  (where  $N_m$  is the number of days in month  $m$ ):

$$SUP1_m - N_m \text{ DAYMX1} \leq 0 \quad (15)$$

$$SUP2_m - N_m \text{ DAYMX2} \leq 0 \quad (16)$$

The take-or-pay clause of Panhandle makes it necessary to introduce a new monthly variable,  $SUPV_m$ , equal to the highest of (1) the actual monthly supply  $SUP2_m$  and (2) 75% of the monthly equivalent of the daily demand contract.

The following monthly constraints ensure the endogenous determination of

$SUPV_m$ :

$$SUPV_m - SUP2_m \geq 0 \quad (17)$$

$$SUPV_m - 0.75 * N_m * \text{DAYMX2} \geq 0 \quad (18)$$

The total annual cost of supply from Consolidated, CTS1, includes commodity, winter requirement, and demand costs, with:

$$CTS1 = \left[ \sum_{m=1}^{12} CC1 * SUP1_m \right] + 12 * \left[ \sum_{m=8}^{12} WRC * SUP1_m \right] + 12 * DC1 * \text{DAYMX1} \quad (19)$$

The total annual cost of supply from Panhandle, CTS2, includes commodity and demand costs with:

$$CTS2 = \left[ \sum_{m=1}^{12} CC2 * SUPV_m \right] + 12 * DC2 * DAYMX2 \quad (20)$$

In order to minimize operating costs and maximize the utilization of their production capacity, natural gas producers generally sell gas at a constant rate and impose heavy penalties (similar to the take-or-pay clauses) on unsteady purchasers. It is therefore assumed that monthly well-head and field-line purchases, respectively SUPWH and SUPFL, are constant throughout the year and limited by maximum production capacities SUPWHT and SUPFLT. Hence the constraints:

$$SUPWH \leq SUPWHT \quad (21)$$

$$SUPFL \leq SUPFLT \quad (22)$$

If CWH and CFL are the average unit costs of well-head and field-line purchases, the corresponding total annual cost is:

$$CTWF = 12 * [CWH * SUPWH + CFL * SUPFL] \quad (23)$$

The pipeline rates and unit costs used in the model are those in effect in 1977, with: CC1 = 1202.4; DC1 = 980.0; WRC = 8.075; CC2 = 1009.2; DC2 = 1860.0; CWH = 787.0; CFL = 1481.0 (\$/MMCF).

### 5.2.3. Gas production modeling

The decision variables related to the EOGC natural gas production system include: (1) the monthly production levels  $PR_m$ , and (2) the monthly production capacity expansion DPRO. Several constraints bear on these production variables. First, it is assumed that the EOGC is constrained to supply a share of 10% of the new gas demand DDGT with its own-produced gas. Such a constraint was actually imposed by the Public Utilities Commission of Ohio (PUCO) in 1978, when the EOGC applied for a relief order from the then existing moratorium on new hook-ups. It follows that:

$$\sum_{m=1}^{12} PR_m \geq 0.1 * DDGT \quad (24)$$

If PROC is the monthly production capacity before expansion, the constraints on actual monthly productions are:

$$PR_m - DPRO \leq PROC \quad (25)$$

Finally, the expansion of productive capacity is limited by the availability of recoverable gas deposits. If the maximum additional production capacity is DPROM, it follows that:

$$DPRO \leq DPROM \quad (26)$$

If CIP is the annualized production capacity unit cost and COMP the production operating unit cost, the total annual production cost is:

$$CTP = CIP * DPRO + \sum_{m=1}^{12} COMP * PR_m \quad (27)$$

The existing monthly production capacity PROC was estimated by dividing by 12 the 1975 historical maximum annual production, with:  $PROC = 11,372/12 = 947.67$  MMCF/month. The unit costs CIP and COMP were estimated for 1977. The total production operating cost in 1977 amounted to \$5,711,000, and the quantity of gas produced to 6200 MMCF, hence:  $COMP = 5,711,000/6200 = 921.129$  \$/MMCF. The 1977 historical (or book) value of the production plant amounted to \$73,299,000. In view of the fact that the production plant has been started recently, it was assumed that its 1977 replacement value would be equal to 1.5 times its historical value, or \$109,948,500. The replacement cost per unit of monthly production capacity (PROC) is then equal to 116,020.22 \$/(MMCF/month). The corresponding annualized figure was computed while assuming (1) an investment lifetime of 30 years, and (2) an interest rate of 12%. The annuity factor turns out to be equal to 0.1241, hence:  $CIP = 116,020.22 * 0.1241 = 14,398.11$  \$/(MMCF/month).



#### 5.2.4 Gas storage modeling

The EOGC storage system is modeled as in Guldman (1983), to which the reader is referred for more details. Hence, only a summary is presented here.

The maximum monthly storage injections or withdrawals depend upon the amount of gas stored, i.e., the reservoir pressure. These maximum flows are estimated as linear functions of the storage saturation rate,  $RSTOR_m$ , a proxy for storage pressure defined as:

$$RSTOR_m = GSTOR_m / STC \quad (28)$$

where  $GSTOR_m$  is the amount of gas in storage at the beginning of month  $m$ , and  $STC$  is the certified storage capacity (i.e., the reservoir capacity for a standard gas pressure). It has been observed, historically, that  $RSTOR_m$  is comprised between a minimum and a maximum saturation rate,  $R_{min}$  ( $= .77$ ) and  $R_{max}$  ( $= 1.18$ ). If  $GINST_m$  and  $GOUST_m$  are the actual injections and withdrawals during month  $m$ , then it follows that:

$$GINST_m \leq A_1 * RSTOR_m + B_1 \quad (29)$$

$$GOUST_m \leq A_2 * RSTOR_m + B_2 \quad (30)$$

$$R_{min} \leq RSTOR_m \leq R_{max} \quad (31)$$

If  $GSTOR_0$  is the non-withdrawable gas necessary to establish minimum pressure conditions, it follows that:

$$RSTOR_m = [GSTOR_0 + \sum_{\mu=1}^{m-1} (GINST_{\mu} - GOUST_{\mu})] / STC \quad (32)$$

The coefficients  $A_1$ ,  $B_1$ ,  $A_2$ ,  $B_2$  in Equations (29) and (30) are assumed to be linear functions of the total storage capacity,  $STC$ , which is defined as the sum of (1) the existing EOGC storage capacity  $STCO$  ( $= 147,594.1$  MMCF in 1977), and (2) the additional storage capacity  $DSTC$ . For instance:

$$A_1 = (\text{STCO} + \text{DSTC}) * A_{10} \quad (33)$$

Constraints (29)-(31) are then rewritten as follows:

$$\begin{aligned} \text{GINST}_m - A_{10} * \sum_{\mu=1}^{m-1} (\text{GINST}_\mu - \text{GOUST}_\mu) - (A_{10} * R_{\min} + B_{10}) * \text{DSTC} \leq \\ (A_{10} * R_{\min} + B_{10}) * \text{STCO} : \text{maximum injection} \end{aligned} \quad (34)$$

$$\begin{aligned} \text{GOUST}_m - A_{20} * \sum_{\mu=1}^{m-1} (\text{GINST}_\mu - \text{GOUST}_\mu) - (A_{20} * R_{\min} + B_{20}) * \text{DSTC} \leq \\ (A_{20} * R_{\min} + B_{20}) * \text{STCO} : \text{maximum withdrawal} \end{aligned} \quad (35)$$

$$\begin{aligned} \sum_{\mu=1}^m (\text{GINST}_\mu - \text{GOUST}_\mu) - (R_{\max} - R_{\min}) * \text{DSTC} \leq (R_{\max} - R_{\min}) * \text{STCO} : \\ \text{Maximum saturation rate} \end{aligned} \quad (36)$$

$$\sum_{\mu=1}^m (\text{GINST}_\mu - \text{GOUST}_\mu) \geq 0 : \text{minimum saturation rate} \quad (37)$$

with:  $A_{10} = -0.07766852$ ;  $B_{10} = 0.14043129$ ;  $A_{20} = 0.15244512$ ;  $B_{20} = -0.06656770$ .

In addition to the monthly storage operations constraints, there is a limit DSTCM to the additional storage capacity, determined by the local availability of natural underground reservoirs (depleted gas deposits or aquifers), hence the constraint:

$$\text{DSTC} \leq \text{DSTCM} \quad (38)$$

The annualized capital cost of new storage per unit of capacity, CIST, has been taken equal to 50 \$/MMCF, a figure consistent with the Federal Power Commission National Gas Survey (1975) average estimate of 57.0 \$/MMCF. The EOGC 1977 average storage operation and maintenance cost per unit of storage flow has been selected, with:  $CS = 33.23$  \$/MMCF. The total storage investment and operation and maintenance cost, CTS, is finally:

$$\text{CTS} = \text{CIST} * \text{DSTC} + \text{CS} * \sum_{m=1}^{12} (\text{GINST}_m + \text{GOUST}_m) \quad (39)$$

### 5.2.5 Gas transmission modeling

The EOGC transmission mains convey gas from the points of connection with the suppliers to the distribution networks of the various communities served by the company. Many important transmission mains do so while passing through the EOGC storage system, as illustrated in Figure 2. Abstracting from the spatial complexities of the system's network, the transmission system is decomposed into two components: (1)  $T_1$ , conveying gas from the suppliers to the storage areas and to the end-use customers, and (2)  $T_2$ , conveying gas from the storage areas and the suppliers to the end-use customers. This simplification of the system is illustrated in Figure 3. Clearly, then, the capacity of  $T_1$  is determined by the peak purchases, while the capacity of  $T_2$  is determined by the peak sales to the end-use customers. The peak monthly sales are exogenous to the costs minimization submodel, and only vary when rates are iteratively readjusted. On the other side, the peak monthly purchases are endogenously determined in the costs minimization submodel and may be reduced by increasing the available storage capacity. Obviously, there is a cost trade-off between the additional transmission and storage capacities, which must be accounted for.

Although it is possible that some excess capacity exists in the transmission component  $T_2$ , no data were available to assess the extent of this excess capacity, which was assumed negligible. The existing capacity of component  $T_2$  was therefore assumed to be equal to the 30-year average January load of the existing customers, as computed with Equations (7)-(9), with  $PT_{20} = 58,620.25$  MMCF/month. The peak daily purchases have taken place on February 1, 1971, when the balance between sales and storage withdrawal/

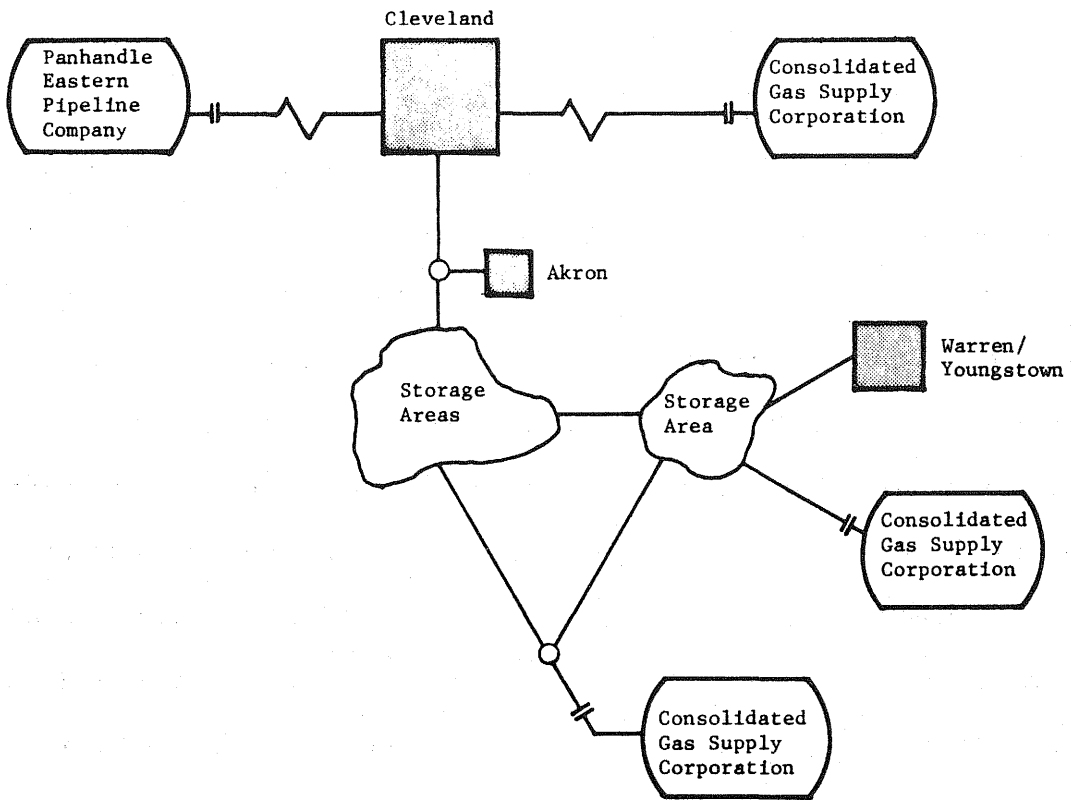


Figure 2 Diagrammatic Representation of the EOGC System

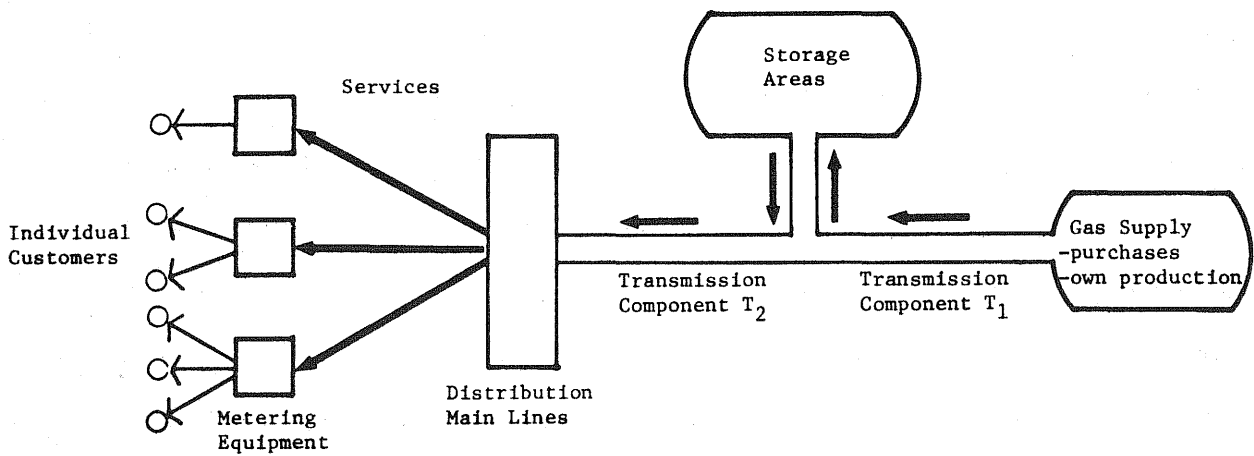


Figure 3 EOGC System Approximation

injection was at a maximum 1770.1 MMCF. This figure would correspond to a monthly purchase rate of 54,873.1 MMCF. In the following, the existing monthly capacity of  $T_1$  is assumed to be:  $PT_{10} = 55,000$  MMCF. The expansion of component  $T_2$  is analyzed in Section 5.3. Here, the only decision variable is the additional transmission capacity  $DPT_1$  for component  $T_1$ . The augmented capacity is the upper limit to monthly transmission flows, hence the constraints:

$$SUP1_m + SUP2_m + SUPWH + SUPFL + PR_m - DPT_1 \leq PT_{10} \quad (40)$$

The 1977 historical (or book) value of the transmission plant amounted to \$102,837,912. In view of the age of the system, it was assumed that the 1977 replacement value of this plant would be equal to 2.5 times its historical value, or \$257,094,785. In addition, it was assumed that (1) component  $T_1$  represents 40% of this investment and component  $T_2$  the remainder, (2) the lifetime of a transmission investment is 30 years, and (3) the discount rate is 12%. The annualized unit expansion costs of the transmission components  $T_1$  and  $T_2$  are then computed as follows:

$$CIPT_1 = (0.4 \times 0.1241 \times 257,094,785) / 55,000 = 232.0397 \text{ \$/ (MMCF/month)}$$

$$CIPT_2 = (0.6 \times 0.1241 \times 257,094,785) / 58,620.25 = 326.5642 \text{ \$/ (MMCF/month)}$$

Cost calculations related to component  $T_2$  are described in the distribution plant submodel section. In the case of component  $T_1$ , the annualized transmission capacity expansion cost is:

$$CTPT_1 = CIPT_1 * DPT_1 \quad (41)$$

The transmission operating costs are considered later, together with the distribution and other operating costs, and are taken proportional to the end-use sales.

### 5.2.6. Gas balance modeling

The loads computed in the monthly load submodel must always be satisfied, hence the monthly supply-demand equality constraints:

$$\text{SUP1}_m + \text{SUP2}_m + \text{SUPWH} + \text{SUPFL} + \text{PR}_m - \text{GINST}_m + \text{GOUST}_m = \text{DGT}_m \quad (42)$$

with:

$$\text{DGT}_m = \text{DGR}_m + \text{DGC}_m + \text{DGI}_m \quad (43)$$

The shadow prices of constraints (42) are noted  $\text{MC}_m$ . They are precisely equal to the marginal costs incurred by an increase of one unit of demand during any month  $m$ . Note, however, that these marginal costs refer only to the costs considered in the linear program (supply, production operations and investment, storage operations and investment, and transmission investment). Therefore, they do not constitute the total marginal costs relevant to marginal cost pricing policy, and will be complemented by other investment and operations marginal costs later.

### 5.3. The New Distribution Plant Submodel

The procedure for determining the additional capacity of the transmission component  $T_1$  necessary to accommodate the peak monthly purchases is endogenous to the cost minimization submodel and was described in Section 5.2.5.

Procedures are proposed here to determine the additional capacities for

(1) the transmission component  $T_2$ , and (2) the distribution system, which must both be able to accommodate the peak monthly end-use load. Common inputs to these procedures are (1) the peak load month  $m_p$ , and (2) the corresponding load  $\text{DGT}_{m_p}$ , as determined in the monthly load submodel.

In the case of the transmission component  $T_2$ , if the peak load  $\text{DGT}_{m_p}$  is smaller than the existing transmission capacity  $\text{PT}_{20}$  (= 58,620.25 MMCF/

month), then there is no need for expanding component  $T_2$ , and the corresponding marginal capacity cost,  $CMPT_2$ , and present value of the additional plant,  $NPT_2$ , are both equal to zero. In the other case, it follows that:

$$CMPT_2 = \begin{cases} CIPT_2 = 326.5642 \text{ \$/MMCF during month } m_p \\ 0 \text{ during all the other months} \end{cases} \quad (44)$$

and

$$NPT_2 = CIPT_2 * (DGT_{m_p} - PT_{20}) / CRF \quad (45)$$

where CRF is the annuity factor (= 0.1241).

It is assumed that the expansion of the end-use load, as measured by the growth rates RMR, RMC, and RMI, is due to the hook-up of new customers. The impact of these connections on the distribution system is twofold. First, costs directly related to the provision of gas service to these new customers are incurred, including local main extensions, services, meters, and land rights costs. The magnitude of such investments is mainly a function of the number of customers, and much less so of their loads, hence these costs are usually referred to as customer costs. Second, in addition to the previous localized costs, the attachment of new loads may require expanding the capacity of some major trunk mains through which most of the community load is conveyed. A major issue, still debated in the regulatory community, is the sharing of total distribution costs among capacity and customer costs. The approach selected here is based on the results of econometric analyses of distribution plant costs at the community level, where these costs are explained by such variables as sales, numbers of customers, population density, etc. These analyses were applied to data obtained from six different distribution utilities, including the EOGC, and are presented in Guldman (1981b, 1982). In the case of the EOGC, the

results imply that 83.3% of the total additional distribution plant costs are customer-related, and 16.7% are capacity-expansion-related. These shares are used in the present analysis as follows. The historical value of the EOGC distribution plant amounted to \$372,284,403 in 1977. On the basis of data provided by EOGC management, the 1977 replacement value of this plant was taken equal to 2.5 times its historical value, or \$930,711,000. The customer-related component is valued at 83.3% of the previous figure, or \$775,282,260, and the capacity-related component at \$155,428,740. The magnitude of the latter value is related to the peak monthly load of the 1977 existing customers  $PD_0$  (= 58,620.25 MMCF), hence the annualized capacity expansion unit cost  $CIPD_1$  is computed as follows:  $CIPD_1 = (0.1241 * 155,428,740) / 58,620.25 = 329.045 \text{ \$/ (MMCF/month)}$ . If the peak load  $DGT_{m_p}$  is smaller than the existing distribution capacity  $PD_0$ , then there is no need for increasing the capacity of the distribution system, and the corresponding marginal capacity cost,  $CMPD_1$ , and present value of the additional capacity,  $NPD_1$ , are both equal to zero. In the other case, it follows that:

$$CMPD_1 = \begin{cases} CIPD_1 = 329.045 \text{ \$/MMCF during the month } m_p \\ 0 \text{ during all the other months} \end{cases} \quad (46)$$

and

$$NPD_1 = CIPD_1 * (DGT_{m_p} - PD_0) / CRF \quad (47)$$

It obviously costs more to connect a huge industrial customer than to connect a residential one, and customer costs must be related to customer sizes as measured by their annual loads. In the absence of more data, it is assumed that these costs are proportional to customer size, hence that the unit customer cost per MCF is the same for all the end-use sectors. The total annual load of the 1977 existing customers, as computed with Equations (7)-(9),



is equal to 399,692.49 MMCF. The average customer cost per MCF,  $CIPD_2$ , assumed equal to the marginal customer cost  $CMPD_2$ , is computed as follows:  $CIPD_2 = (0.1241 * 775,282,260) / 399,692.49 = 240.7164 \text{ \$/MMCF}$ . The present value of the customer-related additional distribution plant,  $NPD_2$ , is then computed as follows:

$$NPD_2 = [RMR * (\sum_{m=1}^{12} DGR_m^\circ) + RMC * (\sum_{m=1}^{12} DGC_m^\circ) + RMI * (\sum_{m=1}^{12} DGI_m^\circ)] * CIPD_2 / CRF \quad (48)$$

where  $DGR_m^\circ$ ,  $DGC_m^\circ$ , and  $DGI_m^\circ$  are computed with Equations (7)-(9).

#### 5.4. The Financial Analysis Submodel

This submodel very much replicates the main calculations that are typically performed prior to regular rate case proceedings, which take place when the utility requests a change in its retail prices in order to be able to achieve the rate of return on the net value of its plant in service (or rate base), as allowed by the state regulatory authorities. Several equations used in this analysis have been developed in Guldman and Czamanski (1980), to which the reader is referred for more details.

The first part of the analysis consists in determining the net plant in service (rate base) and the depreciation expense. It is assumed that the whole new plant is put in service in the same single period (i.e., within a year's time), and that the market growth takes place in a similar way. Of course, this is an approximation of reality, wherein the growth in both plant and market takes place progressively. However, such an approximation is acceptable in view of the purpose of the model, i.e., a general evaluation of marginal cost pricing policy. The total cost CT minimized in the costs minimization submodel includes both operating and annualized investment costs,

noted  $OMC_1$  and  $PIS_1$ , respectively. The investment costs include the production, storage and transmission capacity costs, with:

$$PIS_1 = CIP * DPRO + CIST * DSTC + CIPT_1 * DPT_1 \quad (49)$$

The present value of this plant is then:

$$NEWPIS_1 = PIS_1 / CRF \quad (50)$$

The operating costs are, of course, equal to:

$$OMC_1 = CT - PIS_1 \quad (51)$$

The next step is to compute the present value of the total new plant,  $NEWPIS$ , including the transmission component  $T_2$  and the distribution system, with:

$$NEWPIS = NEWPIS_1 + NPT_2 + NPD_1 + NPD_2 \quad (52)$$

The calculations of (1) the total plant in service, sum of the initial plant in service (= \$617,338,511), of the replacement plant, and of the new plant, (2) the depreciation expense  $DEPEXP$ , (3) the accumulated provision for depreciation  $TAPD$ , and (4) the net plant in service or rate base,  $NETPIS$ , are the same as those described in Guldman and Czamanski (1980).

The second part of the analysis consists in determining the revenue from gas sales,  $X$ , which enables the utility to earn the allowed rate of return on its rate base. It is assumed that this rate of return is equal to 12.06% (1978 value prescribed by the Public Utilities Commission of Ohio). The allowed operating income,  $AOPINC$ , is then:

$$AOPINC = 0.1206 * NETPIS \quad (53)$$

The actual operating expenses of the utility,  $ACOPEX$ , are the sum of the operating and depreciation expenses. The operating expenses include:

- (a) the operating costs  $OMC_1$ , determined in the costs minimization submodel and
- (b) the other operating costs  $OMC_2$ , not considered previously and assumed proportional to total gas sales (i.e., transmission and distribution operations,

customer services, and administration costs), with a unit cost  $COM_2$  determined with 1977 data and equal to 209.48495 \$/MMCF. If DGTT is the total annual load, then:

$$OMC_2 = 209.48495 * DGTT \quad (54)$$

It follows that:

$$ACOPEX = OMC_1 + OMC_2 + DEPEXP \quad (55)$$

The total operating revenues, TOPREV, are the sum of the revenues from gas sales, XE, and of other revenues derived from the transportation of gas of others and from non-utility operations such as building rentals. These other revenues are empirically related to the total plant in service, TOTPIS. The total operating revenues are then:

$$TOPREV = XE + 0.005263 * TOTPIS \quad (56)$$

In order to determine the net operating income NOPINC, it is necessary to account for several taxes such as federal income, revenue, property, and payroll taxes, and for deductions related to liberalized depreciation, interest charges, and investment tax credits. These calculations are described in detail in Guldman and Czamanski (1980). The net operating income NOPINC is then expressed as a linear function of the unknown X, and the fundamental revenue requirement equation

$$NOPINC(XE) = AOPINC \quad (57)$$

is solved to yield the necessary revenues from gas sales, XE. The corresponding average volumetric rate is then:

$$\bar{P} = XE/DGTT \quad (58)$$

### 5.5. The Pricing Submodel

The calculation of the average rate  $\bar{P}$  used in the ACPP block has been described in the previous section. The focus is now on the calculation, for each month  $m$ , of the total marginal costs  $TMC_m$  incurred by a marginal increase of month  $m$  total load  $DGT_m$ , and on the determination of monthly rates based on these marginal costs.

The total marginal cost  $TMC_m$  is the sum of (1) the marginal cost  $MC_m$  as determined by the cost minimization submodel (see Section 5.2), (2) the marginal capacity costs for the transmission component  $T_2$ ,  $CMPT_2$ , and the distribution system,  $CMPD_1$  and  $CMPD_2$  (see Section 5.3), and (3) the other operating marginal costs,  $COM_2$  (see Section 5.4). Although the best way to deal with the distribution plant customer marginal cost would be to design a separate customer charge specifically aimed at recovering these customer-related costs, such a two-part tariff cannot be handled by the proposed demand functions. Therefore, the marginal customer cost  $CMPD_2$  (= 240.7164 \$/MMCF) is considered as applying to all the 12 months. This should lead to a substantial recovery of the corresponding total costs. The operating marginal cost  $COM_2$  (=209.48495 \$/MMCF) is also effective each month. However, as suggested in Equations (44) and (46), the marginal capacity costs  $CMPT_2$  (=326.5642 \$/MMCF) and  $CMPD_1$  (=329.045 \$/MMCF) only apply to the peak-load month  $m_p$ . It follows that:

$$TMC_m = \begin{cases} MC_m + CMPD_2 + COM_2 + CMPT_2 + CMPD_1 & \text{if } m = m_p \\ MC_m + CMPD_2 + COM_2 & \text{if } m \neq m_p \end{cases} \quad (59)$$

According to the theoretical framework presented in Section II, monthly prices should be equated to marginal costs, with:

$$P_m = TMC_m \quad (60)$$

Such a pricing pattern is optimal only if the monthly loads resulting from such prices generate the same marginal costs  $TMC_m$ . However, the magnitude of the peak marginal costs and the price elasticity of the corresponding peak load function may lead to the formation of a new peak, which may in turn cease to be peak when charged the peak marginal costs. This is the well-known shifting-peak case, wherein capacity cannot be justified by the demand in any period alone. In such a case, Steiner (1957) has shown that capacity must be justified by the combined demands in two or more periods, with prices determined in such a way that the demands in these periods are equal, while still higher than those in the other periods. These equal demand periods are those in which the peak-shifting relationships apply, while a firm peak prevails between this subgroup of periods and all the others. The peak marginal capacity costs are spread over the peak-shifting periods so as to lead to equal demands, and hence to a full recovery of these costs. To illustrate in a general fashion the above discussion, assume that the monthly periods are divided in two groups:  $M_1$ , the set of months where the peak-shifting relationship prevails, and  $M_2$ , the set of all the other months. The optimal set of prices should verify the following conditions.

$$P_m = TMC_m \quad m \in M_2 \quad (61)$$

$$\sum_{m \in M_1} P_m = \sum_{m \in M_1} TMC_m \quad (62)$$

$$DGT_m(P_m) = DGT_0 \quad \forall m \in M_1 \quad (63)$$

where  $DGT_0$  is the peak demand effective in all the peak periods. A more complete discussion of the above pricing rules is presented in Appendix B, in connection with the analysis of the model output presented in Section 6.3.

Whether prices have been determined under firm or shifting peak conditions, they may not achieve the revenue requirements objective. In such a case, Baumol and Bradford (1970) have shown that the second-best alternative

is to set the price of each product sold by the utility so that its percentage deviation from marginal cost is inversely proportionate to the product's price elasticity of demand. If all the elasticities are equal, the prices should simply be set proportional to marginal costs. In the present case, the products can be identified with the twelve monthly gas loads. As the monthly elasticities of the commercial and industrial sectors are constant throughout the year and those of the residential sector vary little, a proportional adjustment of the marginal costs is an acceptable approximation of the inverse elasticity rule. The final adjustment factor is determined iteratively by comparing, at the end of each cycle of the MCPP block, the gas sales revenues XE that would be necessary to earn the allowed operating income with the actual gas sales revenues XA based on the prices applied at the beginning of the cycle. The revenue deficit (or surplus) is defined as:

$$DF = XA - XE \quad (64)$$

and the price adjustment factor is then:

$$ADJ = XE/XA \quad (65)$$

#### 5.6. The Evaluation Submodel

Several criteria may be used to compare the relative merits of average cost and marginal cost pricing policies (ACPP and MCPP). Although they are not independent one from the other, these criteria may be grouped into four categories related to: (1) energy conservation, (2) capacity utilization, (3) financial impact, and (4) end-use efficiency.

The impact of a pricing policy on energy conservation is best measured by the total annual gas load DGTT induced by this policy. The extent of capacity utilization is measured by the load factor, equal to the ratio of

the average to the peak daily loads. As a proxy consistent with the temporal disaggregation of the model, a load factor based on the monthly peak load is computed, with:

$$LF = DGT / (12 * DGT_{m,p}) \quad (66)$$

An important financial criterion is the capital requirement for new plant, as measured by NEWPIS (see Section 5.4.). Finally, the end-use efficiency of a pricing policy is measured by consumers' surpluses computed for each month and each sector separately. Consider the typical demand curve  $P(D)$  in Figure 4. The consumer's surplus  $CS_0$  at price  $P_0$  is measured by the shaded area  $S_0$ , or:

$$CS_0 = \int_0^{D_0} P(D) dD - P_0 D_0 \quad (67)$$

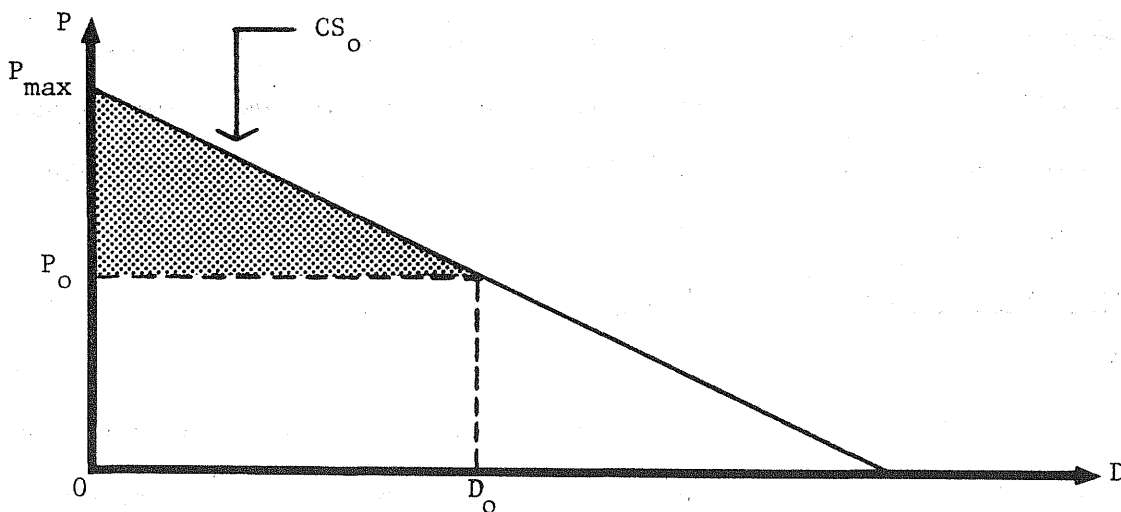


Figure 4 Typical Demand Curve and Consumer's Surplus

In the present study, the monthly demand functions to be used to compute the consumer's surpluses have a constant price elasticity [see Equations (10)-(12)], hence the price  $P_{max}$  corresponding to a zero demand tends toward infinity, leading to an infinite surplus, which is unrealistic. Actually, the demand

functions are of the constant-elasticity type only over a range of prices, which extends neither to zero nor to infinity. In other words, there is a finite price  $P_{\max}$  where the demand falls to zero. However, neither  $P_{\max}$  nor the form of the demand function in the vicinity of  $P_{\max}$  can be ascertained. Total consumers' surpluses for each pricing policy thus cannot be determined. However, for the purpose of comparing average and marginal cost pricing policies, the difference in consumer's surplus between the two policies can be estimated and used to assess their relative merits. If  $(P_1, D_1)$  and  $(P_2, D_2)$  are the equilibrium prices/quantities, for a given month and sector, of the ACPP and MCPP, respectively, then the above difference is computed as:

$$\Delta CS(P_1 \rightarrow P_2) = \int_{D_1}^{D_2} P(D) dD + (P_1 D_1 - P_2 D_2) \quad (68)$$

If  $P_2 < P_1$ , then  $\Delta CS(P_1 \rightarrow P_2) > 0$ , and the MCPP leads to a greater end-use efficiency than does the ACPP. The reverse holds true if  $P_1 < P_2$ . The surplus differentials  $\Delta CS$  are then summed up over all the twelve months as well as over the three sectors, providing sectoral and global assessments of relative end-use efficiencies.

## VI. Application of the Gas Marginal Cost Pricing Model

### 6.1 Assumptions

The reference price  $P_A$  [see Equations ((10)-12)] was determined as the average uniform price providing the revenues required by the existing customers' loads, as computed with Equations (7)-(9). The maximum supplies were selected to reflect the 1977 supply conditions, with: SUP1T = 300,000 MMCF, SUP2T = 60,000 MMCF, SUPWHT = 1000 MMCF, and SUPFLT = 2500 MMCF. No production or storage capacity investment expansions were allowed in the model. No new transmission or distribution investments turned out to be necessary, and, for an annual total load of 399,692 MMCF, the average price turned out to be:  $P_A = 1783.580$  \$/MMCF.



The model was then applied under the assumption of a 50% growth in the numbers of residential, commercial, and industrial customers (i.e.,  $RMR = RMC = RMI = 0.5$ ). The assumed maximum annual supplies from Consolidated and Panhandle reflect the current supply shares of these two companies, with:  $SUP1T = 600,000$  MMCF, and  $SUP2T = 100,000$  MMCF. The assumptions with respect to maximum well-head and field-line purchases also reflect the current supply ratio for these two sources, with:  $SUPWHT = 2000$  MMCF/month, and  $SUPFLT = 5000$  MMCF/month. Finally, the maximum incremental production and storage capacities were set as follows:  $DPROM = 3000$  MMCF/month, and  $DSTCM = 100,000$  MMCF.

## 6.2. Equilibrium Average Cost Pricing Policy

The average cost pricing iterative procedure reached the equilibrium price  $P_e = 1745.180$  \$/MMCF in five iterations, given an error bound of 0.001 MMCF applied to each monthly load and an initial price set equal to  $P_A$ . The uniqueness of this equilibrium price is demonstrated in Appendix A.

The equilibrium monthly sectoral load are presented in Table 3. The residential, commercial, and industrial sectors make up for 47.75%, 19.18%, and 33.07% of the total annual load of 604,561 MMCF. The January load (88,598 MMCF) emerges as a strong peak, clearly dominating the December (81,019 MMCF) and February (79,610 MMCF) loads. All the other months' loads are significantly smaller than these three months' loads.

The optimal supply pattern corresponding to these equilibrium loads is presented in Table 4. The maximum amounts of gas available from Panhandle and from local well-head producers are exhausted in priority. Panhandle supplies are purchased in such a way that the take-or-pay clause (75% of the contract demand) need not be implemented. Well-head gas is purchased in priority because of its low cost (787 \$/MMCF), whereas field-line gas

Table 3 Equilibrium Monthly Loads (MMCF) with Market Growth Rates Equal to 50% - Average Cost Pricing Policy

Month	Residential Load	Commercial Load	Industrial Load	Total Load
April	23,517	9,453	16,585	49,555
May	14,018	5,824	15,310	35,152
June	6,767	3,048	14,334	24,149
July	5,318	2,494	14,139	21,951
August	5,608	2,605	14,178	22,390
September	9,334	4,031	14,679	28,045
October	18,545	7,557	15,919	42,020
November	31,079	12,345	17,602	61,027
December	44,259	17,386	19,374	81,019
January	49,255	19,297	20,046	88,598
February	43,330	17,031	19,249	79,610
March	37,684	14,871	18,490	71,045
Total	288,714	115,942	199,905	604,561

Table 4 Optimal Supply Pattern (MMCF) - Average Cost Pricing Policy

Month	Consolidated SUP1	Panhandle SUP2	Well-Head SUPWH	Production PR	Storage Flow*
April	54,498	7,317	2,000	1,679	-15,940
May	32,257	7,317	2,000	1,679	- 8,101
June	31,248	7,317	2,000	1,679	-18,095
July	27,644	7,317	2,000	1,679	-16,690
August	26,788	7,317	2,000	1,679	-15,394
September	31,247	7,317	2,000	1,679	-14,198
October	44,120	7,317	2,000	1,679	-13,095
November	19,534	9,756	2,000	1,679	+28,057
December	47,150	9,756	2,000	1,679	+20,433
January	54,498	9,756	2,000	1,679	+20,665
February	48,660	9,756	2,000	1,679	+17,514
March	42,765	9,756	2,000	1,679	+14,844
Total	460,409	100,000	24,000	20,152	0

\*The sign (-) points to deliveries to storage, and the sign (+) to withdrawals from storage

is never purchased because of its high cost (1481 \$/MMCF). Production is not a cost-attractive alternative because of the high cost of production capacity, which is expanded by 731.671 MMCF/month, or just enough to provide for the minimum production requirement (i.e., 10% of the total demand increment equal to 201,520 MMCF). As could be expected, the expanded production capacity is always fully used. The balances of monthly requirements are provided by Consolidated and by the storage system. Additional storage capacity is developed up to the maximum expansion (100,000 MMCF), and this expanded capacity is fully used (101,513 MMCF of total annual deliveries/withdrawals, with a maximum saturation rate equal to 1.18 at the end of October). Total monthly gas purchases reach a January peak of 67,934 MMCF, calling for an expansion of the transmission system  $T_1$  to accommodate an additional monthly flow of 12,934 MMCF.

The total cost CT minimized in the linear program amounts to \$766,472,519, including (1) the operating costs  $OMC_1 = \$747,936,756$ , and (2) the annualized investment costs  $PIS_1 = \$18,535,763$  (for the production, storage, and transmission capacities expansions). The operating costs  $OMC_1$  include the total commodity charges (85.393% of CT), the total demand charges (3.734% of CT), Consolidated's total winter requirement charges (2.688% of CT), and the storage O&M costs (0.880% of CT).

The transmission component  $T_2$  and the distribution system capacities must be expanded from 58,620 MMCF/month to 88,598 MMCF/month, implying total plant investments equal to  $NPT_2 = \$78,885,808$  and  $NPD_1 = \$79,485,056$ . The customer-related additional distribution plant is equal to  $NPD_2 = \$387,640,832$ .

The total new plant amounts to  $NEWPIS = \$695,373,056$ . This value is input to the financial analysis submodel, leading to the revenue requirement from gas sales  $X = \$1,055,068,272$ , and to the equilibrium price  $P_e = 1745.180$

\$/MMCF. The values of the evaluation criteria are presented in Section 6.5, together with those related to the equilibrium marginal cost pricing policy.

### 6.3. Search for the Optimal Marginal Cost Pricing Pattern

The MCPP iterative procedure has been first applied while starting with the marginal costs generated by the ACP equilibrium demand pattern and allocating the marginal capacity distribution costs ( $CMPT_2 + CMPD_2$ ) to the peak monthly load exclusively. This approach generates a cyclical, non-convergent demand pattern, as shown in Table 5. The peak January load in Case A leads

Table 5 Peak-shifting Pattern Under Peak Month Marginal Cost Allocation

Month	Case A			Case B			Case C		
	Load* (MMCF)	MC <sub>m</sub> (\$/MMCF)	TMC <sub>m</sub> (\$/MMCF)	Load (MMCF)	MC <sub>m</sub> (\$/MMCF)	TMC <sub>m</sub> (\$/MMCF)	Load (MMCF)	MC <sub>m</sub> (\$/MMCF)	TMC <sub>m</sub> (\$/MMCF)
April	49,555	1,202.4	1,652.6	50,619	1,202.4	1,652.6	50,619	1,202.4	1,652.6
May	35,152	1,202.4	1,652.6	35,952	1,202.4	1,652.6	35,952	1,202.4	1,652.6
June	24,149	1,202.4	1,652.6	24,786	1,202.4	1,652.6	24,786	1,202.4	1,652.6
July	21,951	1,202.4	1,652.6	22,555	1,202.4	1,652.6	22,555	1,202.4	1,652.6
August	22,390	1,202.4	1,652.6	23,001	1,202.4	1,652.6	23,001	1,202.4	1,652.6
September	28,045	1,202.4	1,652.6	28,739	1,202.4	1,652.6	28,739	1,202.4	1,652.6
October	42,020	1,202.4	1,652.6	42,922	1,202.4	1,652.6	42,922	1,202.4	1,652.6
November	61,027	1,299.3	1,749.5	60,971	1,299.3	1,749.5	60,971	1,299.3	1,749.5
December	81,019	1,299.3	1,749.5	80,948	1,299.3	2,405.1	72,449	1,299.3	1,749.5
January	88,598	1,923.3	3,029.1	73,410	1,299.3	1,749.5	88,522	1,923.3	3,029.1
February	79,610	1,299.3	1,749.5	79,540	1,923.3	2,373.5	71,492	1,299.3	1,749.5
March	71,045	1,299.3	1,749.5	70,982	1,299.3	1,749.5	70,982	1,299.3	1,749.5

\* ACP equilibrium load.

to a peak January marginal cost, which leads to a depressed January load in Case B (73,410 MMCF). The resulting marginal cost pattern involves two peaks, one in December and one in February, leading to the renewed dominance of the January peak in Case C. The load pattern in Case C generates marginal costs identical to those in Case A, hence the cycle.

The nature of the peak-shifting cycle can be further analyzed by considering the L.P. cost minimization dual inequality:

$$MC_m \leq CC1 + \delta_m * 12 * WRC + VS1 + V1X_m + VTRAN_m \quad (69)$$

where (1)  $\delta_m = 1$  for the months of November through March,  $\delta_m = 0$  otherwise; (2) VS1 is the shadow price of the constraint on total annual supplies from Consolidated [see Equation (13)]; (3)  $V1X_m$  is the shadow price of the maximum purchase constraint for month  $m$  from Consolidated [see Equation (15)]; and  $VTRAN_m$  is the shadow price of the transmission flow constraint for month  $m$  [see Equation (40)]. Inequality (69) is actually always an equality because the supply variable  $SUP1_m$  to which it is associated is always positive. The total annual supply from Consolidated is never exhausted, hence VS1 is always equal to zero. In addition, the following dual constraints always hold:

$$\sum_{m=1}^{12} V1X_m = (12/30)DC1 = 392 \text{ (\$/MMCF)} \quad (70)$$

$$\sum_{m=1}^{12} VTRAN_m = CIPT_1 = 232.04 \text{ (\$/MMCF)} \quad (71)$$

The total monthly transmission flow is the sum of the monthly purchases and EOGC own-production. As the pattern of purchases from Panhandle and wellhead producers does not vary (these are high priority sources), and as gas is produced by the EOGC at a constant monthly level, it is clear that the maximum monthly purchases from Consolidated and the maximum monthly transmission flows are taking place at the same time. If the peak transmission month is unique, then:

$$MC_m = 1202.4 + \delta_m * 96.9 + \gamma_m * 624.0 \quad (72)$$

where  $\gamma_m = 1$  if  $m$  is the peak transmission month,  $\gamma_m = 0$  otherwise. The other operating and customer marginal costs ( $COM_2 + CMPD_2 = 450.2$  \$/MMCF) apply in each month. If  $\beta_m = 1$  for the peak load month and  $\beta_m = 0$  in the other months, the total marginal cost function  $TMC_m$  [see Equation (59)] can be written as:

$$TMC_m = \begin{cases} 1,652.6 + \gamma_m * 624.0 + \beta_m * 450.2 & (m=1 \rightarrow 7) \\ 1,749.5 + \gamma_m * 624.0 + \beta_m * 450.2 & (m=8 \rightarrow 12) \end{cases} \quad (73)$$

Equation (73) fully explains the pattern of marginal costs observed in Table 5: in Cases A and C, both peak marginal costs are assigned to January (i.e.,  $\gamma_{10} = \beta_{10} = 1$ ), whereas in Case B the peak transmission/purchase marginal cost is assigned to February (i.e.,  $\gamma_{11} = 1$ ) and the peak distribution marginal cost is assigned to December (i.e.,  $\beta_9 = 1$ ).

The load patterns resulting from the pricing patterns implied by Equation (60) and (73) were calculated for all the feasible combinations of values of the parameters  $\beta_m$  and  $\gamma_m$ . As could be expected, none of these load patterns lead to marginal costs ( $TMC_m$ ) equal to the initial prices. Therefore, in order to obtain a sustainable marginal-cost-based pricing pattern, it is necessary to spread the peak marginal capacity costs over several months, leading to equal peak loads for these months. Two inter-related issues then arise: (1) over how many and which months should these marginal capacity costs be spread, and (2) should both marginal costs or only the distribution-related one be spread over these months? The selection of the appropriate pricing rule is analyzed in detail in Appendix B. From a practical viewpoint, we have considered the following a priori feasible apportionments over the three highest loads winter months (December, January, and February): (1) allocate both marginal costs to any combination of two out of these three months; (2) allocate both marginal costs to the three months; (3) allocate the marginal distribution capacity cost to any combination of two out of the three months, and allocate the marginal transmission/purchase cost wholly to the third month; (4) allocate the marginal distribution capacity cost to the three months, and the transmission/purchase one to any of the remaining nine months, alternatively. The prices to be applied in the equal peak load months under the above possible allocations were determined by solving a set of non-linear equations with Newton's method [see, for instance, Ortega and Rheinboldt (1970), pp. 181]. Assuming there are

n ( $j=1 \rightarrow n$ ) such peak months, and denoting  $P_j$  the price in month  $j$  and  $x$  the equal peak load, the set of equations to be solved is:

$$DGR_j(P_j) + DGC_j(P_j) + DGL_j(P_j) - x = 0 \quad (j=1 \rightarrow n) \quad (74)$$

$$\sum_{j=1}^n P_j = n*1749.5 + 450.2 + \alpha*624.0 \quad (75)$$

where  $\alpha=1$  if the peak transmission/purchase marginal cost is apportioned over these peak load months, and  $\alpha=0$  otherwise. The results show that the only acceptable pricing pattern involves the apportionment of both marginal capacity costs over the three winter months. This pricing pattern and the corresponding loads, presented in Table 6, were then iteratively adjusted to satisfy the revenue requirement constraint, while, of course, maintaining the equality of the peak months loads.

#### 6.4. Equilibrium Marginal-Cost-Based Pricing Policy

Starting from the pricing pattern obtained as described in the previous section, the equilibrium pricing pattern satisfying the revenue requirement constraint was determined in four iterations. Both the initial and final price/load patterns are presented in Table 6. The initial pattern leads to gas sales revenues  $XA = \$1,112,202,721$  for a corresponding maximum allowed revenue  $XE = \$1,026,770,608$ , and therefore to an excess revenue of  $\$85,432,113$  for the utility. To correct for this difference, monthly prices are adjusted downward by a multiplier equal to 0.92194 (this multiplier applies to the sum of the December, January, and February prices). As a consequence, the maximum monthly load increases from 77,144 MCF to 79,346 MMCF (or by 2.85%).

The optimal supply pattern corresponding to the equilibrium is presented in Table 7. As in the case of the equilibrium ACP policy (see Table 4), the supplies available from Panhandle and local producers are exhausted in priority, gas is produced by the EOGC at the minimum feasible level, and

Table 6 Initial and Equilibrium Price/Load Patterns in the Case of the Marginal-Cost-Based Pricing Policy

Month	Initial Pattern		Final Equilibrium Pattern				
	Price (\$/MMCF)	Total Load (MMCF)	Price (\$/MMCF)	Residential Load (MMCF)	Commercial Load (MMCF)	Industrial Load (MMCF)	Total Load (MMCF)
April	1,652.6	50,619	1,523.6	24,296	9,872	18,091	52,259
May	1,652.6	35,952	1,523.6	14,404	6,083	16,700	37,187
June	1,652.6	24,786	1,523.6	6,953	3,184	15,635	25,772
July	1,652.6	22,555	1,523.6	5,464	2,604	15,423	23,491
August	1,652.6	23,001	1,523.6	5,762	2,720	15,465	23,947
September	1,652.6	28,739	1,523.6	9,591	4,210	16,012	29,841
October	1,652.6	42,922	1,523.6	19,055	7,893	17,365	44,312
November	1,749.5	60,971	1,612.9	31,673	12,660	18,512	62,846
December	2,006.7	77,144	1,851.8	43,634	17,059	18,653	79,346
January	2,613.4	77,144	2,405.2	45,606	17,415	16,325	79,346
February	1,908.0	77,144	1,761.7	43,232	16,980	19,134	79,346
March	1,749.5	70,982	1,612.9	38,403	15,251	19,446	73,101

Table 7 Optimal Supply Pattern (MMCF) - Marginal Cost Pricing Policy

Month	Consolidated SUP1	Panhandle SUP2	Wellhead SUPWH	Production PR	Storage Flow
April	48,379	7,317	2,000	1,697	- 7,133
May	45,582	7,317	2,000	1,697	-19,409
June	30,353	7,317	2,000	1,697	-15,595
July	29,168	7,317	2,000	1,697	-16,690
August	28,327	7,317	2,000	1,697	-15,394
September	32,998	7,317	2,000	1,697	-14,198
October	46,393	7,317	2,000	1,697	-13,095
November	25,284	9,576	2,000	1,697	+24,108
December	41,512	9,576	2,000	1,697	+24,382
January	45,228	9,576	2,000	1,697	+20,665
February	48,379	9,576	2,000	1,697	+17,514
March	44,803	9,576	2,000	1,697	+14,844
Total	466,406	100,000	24,000	20,359	0



Consolidated is, together with the storage system, the marginal supplier. It is important to note that, although the purchases from Consolidated peak at the same level in April and February, the April peak is insensitive to an increase in the load in April or any other month [the shadow price of constraint (15) for April is equal to zero]. Indeed, an increase in demand in April would be supplied by injecting less gas in storage during that month, and increasing injections in later months. The peak February purchases take place during the peak demand period, and hence is directly a function of this peak demand (= 79,346 MMCF). This confirms the appropriateness of the selected allocation rule. The storage system is developed up to the maximum capacity, and is fully used, as in the case of the ACP policy. The other characteristics of the MCP equilibrium are presented in the next section, when compared with those of the ACP equilibrium.

#### 6.5. Comparative Evaluation of the Average and Marginal Costs Pricing Policies

The values of various criteria measuring the performances of both pricing policies are presented in Table 8. These criteria are regrouped into four categories related to end-use load, supply, investment and finances, and consumer's and producer's surpluses.

The load-related criteria indicate very slight decreases in total annual residential and commercial sales when shifting from the ACP to the MCP. These decreases are compensated by a 3.43% increase in annual industrial sales, leading to an overall total annual sales increase of 6,205 MMCF (or 1.03%). The industrial sales shift is due to the fact that the reduction in the winter months sales is more than compensated by increased demands during the summer months, because of both high summer loads and the longer duration of the summer season. However, the peak month sales decrease significantly by 9,252 MMCF (or -10.44%) when shifting from the ACP to the MCP.

Table 8 Evaluation Criteria for the Average and Marginal Cost Pricing Policies

	Average Cost Pricing Policy (ACPP)	Marginal Cost Pricing Policy (MCP)
<u>Load-Related Criteria</u>		
Annual Residential Sales (MMCF)	288,714	288,073
Annual Commercial Sales (MMCF)	115,942	115,932
Annual Industrial Sales (MMCF)	199,905	206,761
Total Annual Sales (MMCF)	604,561	610,766
Peak Sales Month	January	December-February
Peak Sales (MMCF)	88,598	79,346
Load Factor (%)	56.86	64.15
<u>Supply-Related Criteria</u>		
Consolidated Daily Demand (MMCF)	1,816.60	1,612.62
Panhandle Daily Demand (MMCF)	325.20	325.20
Production Capacity Expansion (MMCF)	731.67	748.90
Storage Capacity Expansion (MMCF)	100,000	100,000
Transmission T <sub>2</sub> Capacity Expansion (MMCF)	12,934	6,831
Total Commodity Charges (\$)	654,516,021	661,727,337
Total Demand Charges (\$)	28,621,784	26,222,978
Total Winter Requirement Charges (\$)	20,601,715	19,884,519
<u>Investment and Financial Criteria</u>		
Investment in Transmission T <sub>1</sub> (\$)	24,182,837	12,773,122
Investment in Transmission T <sub>2</sub> (\$)	78,885,808	54,538,480
Investment in Distribution D <sub>1</sub> (\$)	79,485,056	54,952,784
Total New Investment (\$)	695,373,056	637,083,136
Rate Base (\$)	1,078,017,020	1,021,140,990
Revenue Requirement (\$)	1,055,068,272	1,057,587,049
<u>Surplus Differentials (\$)</u>		
Residential Market	0	-8,768,365
Commercial Market	0	-2,617,883
Industrial Market	0	+14,971,521
Total Market	0	3,585,273
Total Producer's and Consumer's Surplus	0	+7,746,252

Correlatively, the monthly load factor increases from 56.86% to 64.15%, with a net gain of 7.29%.

Some of the supply-related criteria reflect the above-mentioned changes in end-use loads. The Consolidated daily demand decreases from 1,861.60 MMCF to 1,612.62 MMCF, leading to a \$2,398,806 decrease in total demand charges. Likewise, the reduced winter gas requirements lead to a decrease in the total winter requirement charges. The decreased peak purchases naturally lead to a decreased incremental capacity for transmission component T<sub>1</sub>. However, these cost decreases are slightly more than compensated by the increase in the total commodity charges due to the increased total annual sales.

The decreases in peak purchases and end-use loads are further reflected by the decreases in the present (i.e., not annualized) values of the investments in new transmission and distribution systems, leading to a decrease of \$58,289,920 (or -8.38%) in the total new investment plant (which includes a non-varying customer-related distribution plant valued at \$387,640,832), and therefore to a decrease in the rate base. However, the resulting decrease in the allowed operating income is compensated by the increase in the total commodity charges, leading to a very slight increase of \$2,518,777 (or 0.24%) in the revenue requirement.

Finally, the analysis of the surplus differential [see Equation (68)] shows an overall increase in consumer's surplus of \$3,585,273 per year when shifting from the ACPP to the MCPP. However, this increase is the balance of a significant increase in the industrial surplus and significant and slight decreases in the residential and commercial surpluses, respectively. The industrial shift is due to the fact that the reduction in surplus during the winter months due to higher prices is much more than compensated by an increase in surplus during the summer months due to lower prices. Such a

compensation effect does not take place for the residential and commercial sectors because of their low load factors and hence their significantly lower summer months loads. Finally, the total producer's and consumer's surplus differential was computed by adding to the consumer's surplus differential the difference between gas sales revenues and operating and new investment costs ( $XE - OMC_1 - 0.1241 * NEWPIS$ ). This differential represents, in fact, the increase in the total welfare function that the MCPP aims at maximizing. The overall welfare increases by \$7,746,252, and the sharing of this increase between consumers and producer is 46.28%/53.72%, respectively.

In summary, besides a very slight increase (1.03%) in total annual gas requirements, which may be considered as negative from an overall energy conservation viewpoint, all the other criteria point to the superiority of the MCPP as compared to the ACPP. This superiority is particularly characterized by lesser new plant requirements, better use of capacity, and higher surpluses for both consumers and producers. However, the net increase in consumers' surpluses is achieved at the expense of the residential and commercial customers and to the benefit of the industrial customers.

## VII. Conclusions .

A marginal cost pricing model for gas distribution utilities has been developed, involving the optimization of gas supply, storage, and transmission, and the search for marginal-cost-based equilibrium prices satisfying the revenue requirement regulatory constraint. This model has been calibrated with data characterizing the East Ohio Gas Company. Its application under a given set of assumptions and constraints clearly points out the superiority of the marginal cost pricing policy (MCPP) as compared to the average cost pricing policy.

Several extensions of this study can be suggested. First, the model could be applied under drastically different assumptions related to supplies, unit

costs, demand elasticities, etc. Such sensitivity analyses could help determine under what ranges of conditions the MCPP is significantly superior to the ACPP. Second, the structure of the model itself could be improved and expanded. For instance, a spatialized representation of the utility system might lead to the calculation of location-specific marginal costs, but at the cost of a considerably increased computational complexity. Also, probabilistic considerations could be introduced into the model, with explicit linkage between service reliability and marginal-cost-based prices. Finally, more realistic, econometrically-estimated cost functions reflecting scale effects could be used instead of the simple linear functions applied in this study, with the drawback, however, of introducing non-linearities into the model. Research is currently undertaken on several of these issues and will be reported elsewhere in the near future.

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## Appendix A

### Uniqueness of the Average Cost Pricing Policy Equilibrium Price

The equilibrium price  $P_e$  produced by the ACP iterative procedure is a solution of the revenue requirement equation

$$XA(P) = XE(P) \tag{A.1}$$

where  $XA(P)$  represents the actual gas sales revenues induced by gas price  $P$ , and  $XE(P)$  the revenues required to develop and operate a gas system providing the loads generated by  $P$ . For the equilibrium price  $P_e$  to be unique, a necessary and sufficient condition is that Equation (A.1) has only one solution.

The function  $XA(P)$  can be written as

$$XA(P) = \sum_{m=1}^{12} [P \cdot DGR_m(P) + P \cdot DGC_m(P) + P \cdot DGL_m(P)] \tag{A.2}$$

where  $DGR_m(P)$ ,  $DGC_m(P)$  and  $DGL_m(P)$  are defined by Equations (10)-(12). As all these demand functions are price-inelastic, it follows that  $XA(P)$  is a continuously increasing function of  $P$ . A synthesis of the financial analysis submodel leads to the following formulation of  $XE(P)$ :

$$XE(P) = 1.0432 \cdot OMC_1(P) + 1.0745 \cdot OMC_2(P) + 0.4629 \cdot DIS(P) + 0.4629 \cdot PIS_1(P) + \text{Constant} \tag{A.3}$$

where  $OMC_1(P)$ ,  $OMC_2(P)$  and  $PIS_1(P)$  are defined by Equations (51), (54), and (49), respectively, and  $DIS(P)$  represents the sum of (1) the new transmission plant  $T_2$  [NPT<sub>2</sub> - Equation (45)] and (2) the new distribution plant  $NPD_1$  [Equation (47)]. The function  $OMC_2(P)$  is proportional to the the total annual demand, and therefore is continuously decreasing with  $P$ . The function  $DIS(P)$  is continuously increasing with the peak monthly load. With uniform pricing, the dominance of January as the peak month is maintained at any price level, and



therefore  $DIS(P)$  is continuously decreasing with  $P$ . The analysis of the functions  $OMC_1(P)$  and  $PIS_1(P)$  is more complex, due to possible discontinuities. The total cost

$$CT(P) = OMC_1(P) + PIS_1(P) \quad (A.4)$$

minimized in the L.P. submodel is a continuous and decreasing function of  $P$  (although not continuously differentiable). However, while their sum is continuous, the functions  $OMC_1(P)$  and  $PIS_1(P)$  may not be so. Indeed a lumpy trade-off between capital and operating costs may take place at some price  $P_0$ , with cost changes  $\Delta OMC_1$  and  $\Delta PIS_1$ . Because of the necessary continuity of  $CT(P)$  at  $P_0$ , it follows that

$$\Delta OMC_1 = - \Delta PIS_1 \quad (A.5)$$

The change in  $XE(P)$  at  $P_0$  is then equal to:

$$\Delta XE = (-1.0432 + 0.4629) \Delta PIS_1 = -0.5803 \Delta PIS_1 \quad (A.6)$$

Two cases can then be considered: Case A, with  $\Delta PIS_1 < 0$ , and Case B, with  $\Delta PIS_1 > 0$ . Besides the above-mentioned possible discontinuities, the functions  $OMC_1(P)$  and  $PIS_1(P)$  are continuously decreasing with  $P$ , hence the possible forms of the function  $XE(P)$  depicted in Figure 5 for Cases A and B.

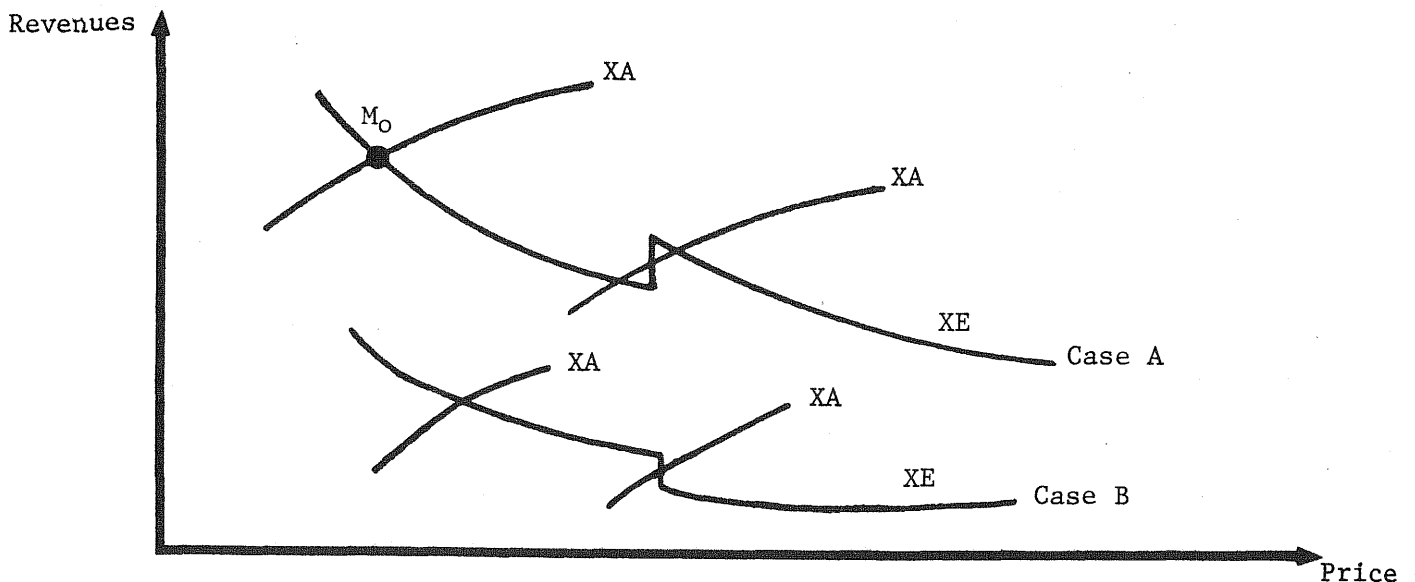


Figure 5 Revenue Functions Configurations

In Case A, depending upon the location of the curve  $XA(P)$ , there might be two or one solutions. In Case B, there might be one or no solution at all. In the present empirical application, a limited sensitivity analysis over the price interval  $[1745.18 - 1815.18]$  clearly shows that the solution  $P_e = 1745.18$  corresponds to point  $M_0$  in Case A, and hence that this equilibrium price is unique. Indeed,  $PIS_1$  decreases continuously with  $P$  within the above interval, with a value of \$17,895,320 at  $P = 1815.18$ . This decrease is related to the decreases in production and transmission capacities, while storage capacity remains at its feasible maximum. As an upper bound case, assume that a total discontinuity takes place at  $P = 1815.18$ , with  $\Delta PIS_1 = -\$17,895,320$ . The required revenue would then increase by  $\Delta XE = \$10,384,654$ . However, at  $P = 1815.18$ , the difference  $(XA - XE)$  is equal to  $(\$1,080,928,200 - \$1,040,119,326) = \$40,862,874$ , hence the impossibility for the two curves  $XA$  and  $XE$  to intersect at any other price but  $P_e$ .

## Appendix B

### Pricing Rules in the Peak-Shifting Case

The pricing rules developed initially by Steiner (1957) are extended by the explicit consideration of both peak purchases and end-use loads costs in the usual welfare function. Assume that there are  $n$  periods ( $i = 1 \rightarrow n$ ), and that the end-use load in period  $i$  is  $X_i$ . Let  $P_i(X_i)$  be the demand function associated to this load. An analysis of the model structure and output shows that the total cost to be considered includes the following four separable components.

$$C_1 = \sum_{i=1}^n C_o * X_i \quad (B.1)$$

$$C_2 = C_2(X_1, X_2, \dots, X_i, \dots, X_n) \quad (B.2)$$

$$C_3 = C_p * Y(X_1, X_2, \dots, X_i, \dots, X_n) \quad (B.3)$$

$$C_4 = C_D * \max_{i=1 \rightarrow n} (X_i) \quad (B.4)$$

$C_1$  includes all the costs that are proportional to sales, i.e., the other operating costs  $OMC_2$  and the customer-related distribution plant costs  $CMPD_2$ .  $C_2$  represents all the costs minimized in the L.P. submodel, at the exclusion of the peak purchase/transmission costs represented by  $C_3$ , where  $Y$  is the peak purchase/transmission flow and  $C_p$  the corresponding unit cost (= 624.0 \$/MMCF). Finally,  $C_4$  represents the peak distribution costs, with  $C_D$  as the corresponding unit cost (= 450.2 \$/MMCF). The objective is to maximize the following welfare function:

$$F(X_1, \dots, X_i, \dots, X_n) = \sum_{i=1}^n \int_0^{X_i} P_i(x) dx - (C_1 + C_2 + C_3 + C_4) \quad (B.5)$$

Assume that the optimal solution vector  $\bar{X} = (\bar{X}_1, \dots, \bar{X}_i, \dots, \bar{X}_n)$  involves the

equality of  $m$  loads, and that the  $X$ 's have been renumbered in such a way that

$$\bar{X}_1 = \bar{X}_2 = \dots = \bar{X}_m > \bar{X}_{m+1} \geq \dots \geq \bar{X}_n \quad (\text{B.6})$$

Let us then replace the variables  $(X_1, X_2, \dots, X_m)$  by a unique variable  $X_o$  in the function  $F$ . We obtain a new function

$$\begin{aligned} G(X_o, X_{m+1}, \dots, X_n) &= \sum_{i=1}^m \int_0^{X_o} P_i(x) dx + \sum_{i=m+1}^n \int_0^{X_i} P_i(x) dx - mC_o X_o \quad (\text{B.7}) \\ &\quad - \sum_{i=m+1}^n C_o X_i - C_2 [X_1(X_o), \dots, X_m(X_o), X_{m+1}, \dots, X_n] \\ &\quad - C_p * Y [X_1(X_o), \dots, X_m(X_o), X_{m+1}, \dots, X_n] - C_D * X_o \end{aligned}$$

The same conditions hold for the optima of  $F$  and  $G$ . At the optimal solution  $(\bar{X}_o, \bar{X}_{m+1}, \dots, \bar{X}_n)$ , the derivatives of  $G$  are equal to zero. It follows that:

$$\begin{aligned} \frac{\partial G}{\partial X_o} \Big|_{\bar{X}} &= \sum_{i=1}^m P_i(\bar{X}_o) - mC_o - \sum_{i=1}^m \frac{\partial C_2}{\partial X_i} * \frac{\partial X_i}{\partial X_o} \Big|_{\bar{X}} \\ &\quad - \sum_{i=1}^m C_p * \frac{\partial Y}{\partial X_i} * \frac{\partial X_i}{\partial X_o} \Big|_{\bar{X}} - C_D = 0 \quad (\text{B.8}) \end{aligned}$$

$$\frac{\partial G}{\partial X_i} \Big|_{\bar{X}} = P_i(\bar{X}_i) - C_o - \frac{\partial C_2}{\partial X_i} \Big|_{\bar{X}} - C_p * \frac{\partial Y}{\partial X_i} \Big|_{\bar{X}} = 0 \quad (i > m) \quad (\text{B.9})$$

Note that by definition

$$\frac{\partial X_i}{\partial X_o} = 1 \quad (i = 1 \rightarrow m) \quad (\text{B.10})$$

and

$$\sum_{i=1}^m \frac{\partial Y}{\partial X_i} * \frac{\partial X_i}{\partial X_o} = \frac{\partial Y}{\partial X_o} \quad (\text{B.11})$$

Two cases can then be considered:

- (a) The peak purchase/transmission flow takes place during the peak load period and varies directly with this uniform peak. Then  $\partial Y / \partial X_o (X) = 1$ , and the pricing rule is:

$$\sum_{i=1}^m P_i(\bar{X}_0) = mC_o + \sum_{i=1}^m \frac{\partial C_2}{\partial X_i} \bigg|_{\bar{X}} + C_p + C_D \quad (\text{B.12})$$

$$P_i(\bar{X}_i) = C_o + \frac{\partial C_2}{\partial X_i} \bigg|_{\bar{X}} \quad (i > m) \quad (\text{B.13})$$

(b) The peak purchase/transmission flow does not take place during the peak load period and is directly related to the end-use load in month  $j \in [m+1, n]$ . Then  $\partial Y / \partial X_o(\bar{X}) = 0$  and  $\partial Y / \partial X_j(\bar{X}) = 1$ , and the pricing rule is:

$$\sum_{i=1}^m P_i(\bar{X}_0) = mC_o + \sum_{i=1}^m \frac{\partial C_2}{\partial X_i} \bigg|_{\bar{X}} + C_D \quad (\text{B.14})$$

$$P_i(\bar{X}_i) = C_o + \frac{\partial C_2}{\partial X_i} \bigg|_{\bar{X}} \quad (i \neq j, i > m) \quad (\text{B.15})$$

$$P_j(\bar{X}_j) = C_o + \frac{\partial C_2}{\partial X_j} \bigg|_{\bar{X}} + C_p \quad (\text{B.16})$$

