Evaluation of Regression Approaches for Predicting Yellow Perch (*Perca flavescens*) Recreational Harvest in Ohio Waters of Lake Erie

QFC Technical Report T2010-01

Prepared for:
Ohio Department of Natural Resources
Division of Wildlife
Lake Erie Fisheries Unit
Sandusky Fisheries Research Unit
305 East Shoreline Drive
Sandusky, OH 44870

Prepared by:
Travis O. Brenden and Weihai Liu
Quantitative Fisheries Center
Department of Fisheries and Wildlife
Michigan State University
153 Giltner Hall
East Lansing, MI 48824

February 9. 2010

Introduction

In Ohio waters of Lake Erie, yellow perch *Perca flavescens* is exploited by both recreational and commercial fisheries, and each fishery is regarded as economically valuable to the state. In 1996, the Ohio Department of Natural Resources (ODNR), Division of Wildlife (DW), along with other state and provincial agencies with management authority on Lake Erie, implemented a quota management system for yellow perch. Under this system, each state and province was annually allocated a percentage of the yellow perch harvest quota based on the amount of lake surface area in their respective jurisdictional waters. It was then left to the states and province as to how their designated quotas would be allocated to commercial and recreational fisheries. In Ohio, the commercial/recreational allocation historically was based upon the percentage of the total harvest that each fishery took during the preceding 5-year period. Such a strategy, however, raised the potential for over- or under-allocation of yellow perch harvest as a result of changes in fishing effort from one year to the next.

Beginning in 2008, the ODNR DW implemented a new quota allocation strategy for Lake Erie yellow perch based upon guidance from Senate Bill 77, the Commercial Fishing Task Force, and the ODNR DW administration. This quota allocation strategy was developed based upon the following guiding principles: 1) maintain healthy stocks of yellow perch in all management units, 2) conduct science-based management of the Lake Erie resources, 3) achieve quota compliance in all management units, 4) implement ODNR DW Policy 2, and 5) reduce commercial fishery bycatch. Policy 2, which was adopted by ODNR DW in 1984, states that

> *"Individual sport fishermen will be accorded the first opportunity to take the harvestable portion of Lake Erie's fish populations. Commercial fishermen may be allocated the remaining portion."*

Under the new ODNR DW allocation strategy, the percentage of Ohio's lakewide yellow perch quota that gets allocated to the commercial fishery is dependent on the predicted status of the yellow perch population. When the abundance of yellow perch in management units (MU) 1-3 (Figure 1) is estimated at greater than 100 million age-2 and older fish, the yellow perch population is considered in Maintenance mode and the commercial fishery is allocated 35% of Ohio's total yellow perch quota. When the yellow perch population is estimated at between 25 and 100 million age-2 and older fish, the population is considered in Conservation mode and the commercial fishery is allocated 30% of Ohio's total yellow perch quota. When the yellow perch population is estimated at less than 25 million age-2 and older fish, the population is considered in Rehabilitation mode, and the commercial fishery is allocated 10% of Ohio's total yellow perch quota. The commercial fishery quota in each MU is then allocated after quota allocation to the recreational fishery components.

The cornerstone of the ODNR FW allocation strategy is the ability to accurately estimate recreational fishery harvest in each of MU prior to the start of the fishing season. In 2008, ODNR DW undertook a modeling evaluation to determine what the best method was for predicting yellow perch recreational fishery harvest in each Lake Erie MU. Briefly, multiple linear regression models were developed that predicted yellow perch recreational fishery harvest or effort based on information pertaining to adult yellow perch and walleye populations in each MU. Information pertaining to walleye population status was incorporated in the modeling evaluation as it was assumed that recreational anglers exhibit "effort switching", in that anglers will switch between targeting walleye and yellow perch depending on the perceived status of the populations. When fitting regression models to yellow perch recreational effort, estimates of harvest were obtained by multiplying predicted effort by mean catch per unit effort of recreational anglers calculated from estimates over the previous three years and mean weight of harvested fish. An approach for predicting yellow perch recreational harvest based on the three-year moving average of yellow perch recreational harvest was also evaluated. Models were chosen using stepwise model selection with significance levels for variable entry and exit set at 0.25 and 0.10. Parameterized multiple regression models were evaluated using several criteria, including coefficients of determination (R^2), patterns in model residuals (deviations between observed and predicted values), and consistency in parameter estimates across management units.

Based on the modeling evaluation conducted by ODNR DW, the approach selected for predicting yellow perch recreational harvest was the one in which recreational effort was the response variable for a relationship with parameters estimated by linear regression, and recreational harvest was estimated by multiplying effort predicted by the regression by mean CPUE and weight of a harvested fish. This indirect approach for predicting yellow perch recreational harvest was chosen for several reasons. It had coefficients of determination that were comparable, albeit generally less, than the model that predicted recreational harvest directly. In terms of residuals, whereas the direct approach tended to have positive residuals at low harvest levels and negative residuals at high harvest levels, the indirect approach tended to have less of a pattern in residuals. Additionally, for the indirect harvest estimation approach there was consistency among the management units in terms of the variables that were selected, with both yellow perch population and walleye population variables occurring in all three MU models. Parameter estimates were also consistent among the MU models for the indirect approach, with higher yellow perch recreational harvest predicted to occur with higher yellow perch abundances and lower harvest predicted to occur with higher walleye abundances.

In Spring 2009, the ODNR DW requested that the Quantitative Fisheries Center (QFC) at Michigan State University review the selected approach for estimating recreational harvest Specifically, the ODNR DW desired answers to the following questions:

1) Is the process for estimating yellow perch harvest by Management Unit prior to the fishing season logical/defensible?
2) Were the analyses that were conducted valid?
3) Are the confidence intervals for estimating the buffered recreational harvest properly developed and applied?
4) Are there better ways to estimate recreational fishery harvest using available information?
5) Can the predictive capabilities of the models (particular in MU2) be improved?

In conducting its review, the ODNR DW asked the QFC to operate under the following assumptions/constraints:

1) Data must be available in late-March as Ohio quota for the upcoming fishing season is allocated by mid-April.
2) Must be relatively simple to explain to stakeholders.
3) Would like to administer it in the spreadsheet format as developed.

Question 1) Is the process for estimating yellow perch harvest by Management Unit prior to the fishing season logical/defensible?

Yes, we believe estimating yellow perch recreational harvest by MU prior to the fishing season in general is logical and defensible. Indeed such analyses are regularly used in a variety of fields to predict a response variable based on models parameterized using historical data (e.g., stock market performance).

Question 2) Were the analyses that were conducted valid?

The validity of an analytic method ultimately stems from the assumptions associated with the method and whether these assumptions are appropriate for the analyzed dataset. By using multiple linear regression to predict yellow perch recreational harvest (or effort) in each MU based on variables pertaining to adult yellow perch and walleye populations, the following assumptions are made.

1) Normality: yellow perch recreational harvest/effort conditioned on predictor variables, is normally distributed. That is the variation in actual harvest or effort about the value

expected given the predictor variables will follow a normal distribution. Note that this is not an inherent assumption of multiple linear regression. Rather, the normality assumption is made for hypothesis testing and the calculation of confidence intervals. Because ODNR DW is using confidence intervals to estimate buffered harvest, a normality assumption is made as part of their analysis.

2) Independence: Conditioned on predictor variables, recreational harvest of yellow perch in one year is independent from harvest in other years.
3) Homoskedasticity: the conditional variance of each year's yellow perch recreational harvest is the same.
4) No measurement error: the adult yellow perch and walleye population variables, which are used to predict yellow perch recreational harvest (or effort), are measured without error.
5) Linearity: there is a linear relationship between yellow perch recreational harvest (or effort) variables and the adult yellow perch and walleye populations (or their transformations).

Although some of these assumptions seem reasonable within context of yellow perch recreational harvest modeling, other assumptions, such as the independence, homoskedasticity, and no measurement error in the independent variables, are more difficult to justify. If yellow perch recreational harvest is higher than expected in one year, then it seems likely that recreational harvest in the next year will also be higher. Alternatively, if yellow perch recreational harvest is lower than expected in one year, then it seems likely that recreational harvest in the next year will also be low. In other words, recreational harvest, even after conditioning based on predictor variables, could be autocorrelated, which is a violation of the independence assumption. It also may be the case that the variability in recreational harvest increases as recreational harvest gets larger, which is a violation of the homoskedasticity assumption. Finally, because the adult yellow perch and walleye population variables that are used as independent variables for predicting yellow perch recreational harvest are in fact themselves predicted quantities, the assumption of no measurement error in the independent variables is also violated.

It is important to note that even if assumptions are violated, this does not necessarily mean that an analysis is not valid. Rather, it simply is important to understand how the analyses may be affected by violation of these assumptions and to take this in consideration when formulating management decisions. Consequences of violating some of the aforementioned assumptions include biased estimation of error variance, invalid inferential test procedures, and poor model fit. Analyses can be modified to account for some of these violations; however, some of these analytic approaches can be complicated and would likely be difficult to explain to stakeholders (Constraint 2) and to implement in a spreadsheet format (Constraint 3). Other, less complicated approaches, can also be used to account for violations of assumptions and we illustrate some of these methods in our answer to questions 4 and 5.

Other aspects of the analyses that were performed by ODNR DW that are worth discussing include the manner by which models were selected (stepwise selection) and how model accuracy was evaluated (R^2). Although sequential algorithms for model selection such as forward, backward, or stepwise selection can be useful for objectively identifying a single model from a potentially very large set of candidate models, one of the shortcomings of these algorithms is that there is no guarantee that the models that are selected are indeed the "best". Model selection approaches can be greatly affected by multicollinearity among dependent variables (Myers 1990). One alternative to using a sequential algorithm to choose among models is to evaluate the performance of the entire set of candidate models (all possible subsets of regression). While all possible subsets of regression historically has been difficult to implement because of computing limitations, the power of today's computers makes these calculations relatively efficient for small (<15 independent variables) datasets. Approaches such as all-subset regressions are sometimes criticized as “data-snooping”, however, this is less of an issue when the main goal of a modeling exercise is to be able to predict an unobserved quantity rather than seeking to understand exactly how each regressor influences a response variable.

Using R^2 to evaluate model performance for predicting yellow perch recreational harvest also may be problematic because this statistic does not truly evaluate the predictive capability of a model in terms of future observations (Myers 1990). R^2 measures how much of the total observed variability in a response variable is explained by a statistical model and is calculated from all the data that has been collected to date. Whether the model that does the best job of predicting all data collected to date will also do the best job of predicting future data collected in the future is not known. There are other approaches that are better at evaluating the future predictive performance of models, such as cross-validation, PRESS statistic, and Mallow’s C_p statistic. In answering Questions 4 and 5, we utilize a more robust approach for evaluating the capacity of models to predict, as of yet, unobserved quantities.

Question 3) Are the confidence intervals for estimating the buffered recreational harvest properly developed and applied?

The approach presently used for estimating buffered recreational harvest is an overly complicated approach, which we believe can be accomplished more simply using standard statistical methods. Based on the description provided by ODNR FW, buffered yellow perch recreational harvest is presently calculated using the following steps:

Step 1 – regress yellow perch recreational harvest (or effort) versus walleye or yellow perch population values.
Step 2 - regress predicted yellow perch recreational harvest (from step 1) on actual harvest.
Step 3 – calculate the upper 100(1-α)% confidence interval limit on the mean response from the regression of step 1.
Step 4 – relate the upper confidence limit to predicted harvest, based on a 2nd order polynomial regression of the upper confidence limit (from step 3) on predicted harvest (step 1).
Step 5 – calculate the buffered yellow perch recreation harvest using the prediction equation from Step 4 applied to the predicted yellow perch recreational harvest from Step 1.

The current procedure appears designed to produce a confidence interval for predictions (i.e., an interval that would enclose the true expected harvest a specified percent of the time), and buffered harvest is simply the upper bound on such an interval. We assume the main goal for using the buffered recreational harvest is to account for the uncertainty in the fitted regression equation to protect yellow perch from overharvest in case the predicted recreational harvest is being underestimated on average. As we explain below, this can be done based solely on the results of step 1 using well established analytic formula. Thus, Steps 2 through 5 are unnecessary. All the information about uncertainty in a specific mean prediction given certain predictors is contained in the residual error variance, the values of predictors used in fitting the regression, and the value of predictors used to generate the estimate of next years harvest. Well established statistical theory allows us to calculate an upper 100(1-α)% confidence limit for the estimated (mean) response at a particular combination of regressor variables using the equation

$$\hat{y}(\mathbf{x}_0)+t_{\alpha/2,n-p}\sqrt{\hat{\sigma}^2\mathbf{x}_0'\left(\mathbf{X}'\mathbf{X}\right)\mathbf{x}_0},$$

where $\hat{y}$ is the estimated mean response calculated from next years predicted walleye and yellow perch population values $\mathbf{x}_0$, $\hat{\sigma}^2$ is the estimated residual variance of the response variables (from the Step 1 regression), $\mathbf{X}$ is the matrix of independent variables (including an intercept dummy variable), also called the design matrix, used in fitting the regression model from Step 1, and $t_{\alpha/2,n-p}$ is the corresponding percentage point of a student's *t*-distribution value with *n-p* degrees of freedom (*n*=number of observations, *p*=number of model parameters including the model intercept) at a particular alpha level. The quantity $\sqrt{\hat{\sigma}^2\mathbf{x}_0'\left(\mathbf{X}'\mathbf{X}\right)\mathbf{x}_0}$ is referred to as the standard error for prediction for the general regression model (Myers 1990). Thus to calculate a buffered yellow perch recreational harvest to account for estimation uncertainty, one simply needs the statistics from the fitted regression model (parameter estimates, mean-squared error of the regression as an estimate of σ^2) and next year's yellow perch and walleye population values.

Many standard statistical software packages provide for the automatic calculation of the standard error of predictions. Unfortunately, this is not the case for the built in EXCEL regression functions or the standard Data Analysis add-in. Consequently, we have developed a simple VBA macro that will fit a multiple linear regression model and calculate confidence intervals (and prediction intervals) for a given set of observations. ODNR FW may find this a useful tool for calculating the buffered recreational harvest of yellow perch for the management units (see "Buffered Harvest Tool.xls" file).

There is reasonably close correspondence between the buffered harvest calculated using the method described by the ODNR FW and the Excel tool that we have developed (Figure 2). Nevertheless, we strongly recommend switching to a standard and accepted approach for generating confidence intervals for predictions as implemented in the Excel tool. The approach is easier to explain and justify, and has known performance characteristics and fewer approximations and assumptions (which might matter in some circumstances).

Finally on this topic, we note that there is a difference between regression confidence intervals for predictions, and prediction intervals. The existing ODNR FW method and the approach we suggest as a replacement both are estimating confidence intervals for predictions. By setting the buffer recreational harvest to the upper bound of such intervals one is adopting a procedure that if used repeatedly should produce values less the true expected value of harvest given the predictor values, a specified proportion of the time. The actual annual recreational harvest, rather than the expected harvest, will exceed the buffer value more often because it is influenced not just by regression uncertainty but also inter-annual variation that is not related to the explanatory variables. If one desired a buffer that accounted for this variation, prediction intervals, rather than confidence intervals, should be used. We have included the calculation of prediction intervals in the Excel tool provided in case ODNR DW chooses to switch to prediction intervals for calculating buffered yellow perch recreational harvest.

Question 4) Are there better ways to estimate recreational fishery harvest using available information?

Question 5) Can the predictive capabilities of the models (particular in MU2) be improved?

We chose to address Questions 4 and 5 together as we believe these questions are inextricably linked. That is, determining whether some method is "better" at estimating yellow perch recreational harvest than what currently is used should be based on whether the new method more accurately predicts recreational harvest. In conducting our evaluation, we abided by the constraints originally provided by ODNR DW:

1) Data must be available in late-March as Ohio quota for the upcoming fishing season is allocated by mid-April.

2) Must be relatively simple to explain to stakeholders.
3) Would like to administer it in the spreadsheet format as developed.

In conducting this evaluation, we followed a similar approach to that of ODNR DW by predicting recreational harvest both directly and indirectly (estimate effort and then predict harvest based on mean CPUE and weight from the preceding 3-yr period). Based on the poor performance of the moving 3-year average approach to estimating recreational harvest that was initially tried by ODNR DW, we chose to not include this approach in our evaluation.

Choice of regressor variables

Because we were not in the position to know what data were available to ODNR DW by the date indicated in Constraint 1, we limited our evaluation to the data provided to us. The one variable that we did add to the candidate set of regressors was the prior year's yellow perch recreational harvest. Including the prior year's recreational harvest as a regressor variable was seen as an intuitively straightforward approach to account for potential autocorrelation in yellow perch recreational harvest. We included the $\log_e$ transformation of the variables provided as well as the original scaling of the variables as potential regressor variables.

Model fitting

Because of Constraints 2 and 3, we limited our analyses to multiple linear regression. Other analytic approaches from the fields of time-series analysis or mixed modeling, which conceivably could improve prediction accuracy of yellow perch recreational harvest, were not attempted because it was felt such approaches would be difficult to explain and to implement in a spreadsheet format.

Rather than using a model-selection algorithm such as stepwise, forward, or backward selection to identify only a single model to evaluate predictive performance, we fit models to all possible combinations of regressor variables. We chose to consider all possible models rather than a subset of models as we wanted to find the best model for predicting future yellow perch recreational harvest and there was no guarantee that we would find this model if we limited consideration to a subset of models.

Evaluation of predictive performance

To evaluate how well each candidate model predicted future yellow perch recreational harvest, we used a data splitting process where we fit each model to a particular time span of the available data (e.g., 1991 – 1999), and then used the fitted models to predict recreational harvest for the next year (e.g., 2000). We then calculated the squared error between predicted and observed yellow perch recreational harvest. We conducted this data splitting and error calculation process for the following time periods:

1) fit model to 1991-1999 data and predict 2000 recreational harvest
2) fit model to 1991-2000 data and predict 2001 recreational harvest
3) fit model to 1991-2001 data and predict 2002 recreational harvest
4) fit model to 1991-2002 data and predict 2003 recreational harvest
5) fit model to 1991-2003 data and predict 2004 recreational harvest
6) fit model to 1991-2004 data and predict 2005 recreational harvest
7) fit model to 1991-2005 data and predict 2006 recreational harvest

Note that we did not include data from 1990 when parameterizing the models as we were not provided the yellow perch recreational harvest and effort values for 1989, which would have affected our ability to fit some of the candidate models. The overall performance of the models was evaluated based on the total prediction error (TPE) of the models, which was calculated by summing the squared deviations between predicted and observed yellow perch recreational harvest (on a $\log_e$ scale) for 2000 to 2006. That is, TPE was calculated as

$$\text{TPE} = \sum_{i=2000}^{2006} \left(y_i - \hat{y}_{i,-i} \right)^2$$

where $\hat{y}_{i,-i}$ is the predicted yellow perch recreational harvest in year *i* (on a $\log_e$ scale) from the regression function fit from all years prior to year *i* and y_i is the observed yellow perch recreational harvest (on a $\log_e$ scale) in year *i.* Conceptually, this measure of the predictive performance of a model is similar to the PRESS statistic, which is widely used as a measure to evaluate prediction performance in regression modeling (Myers 1990).

Results and discussion

In our evaluation, we found the estimating yellow perch harvest directly generally resulted in a lower TPE than when harvest was estimated indirectly (Table 1). The one exception to this was for MU3, where the indirect harvest estimation resulted in a somewhat smaller TPE. The difference in TPE for MU3 was quite a bit smaller than the TPE difference for MU 1 and 2 when harvest was estimated directly, which leads us to recommend that yellow perch recreational harvest be estimated directly.

The models that performed best in predicting yellow perch recreational harvest from our evaluation included more regressor variables than those currently used by ODNR DW (Tables 1 and 2). However, the fact that these more complex models had much smaller TPEs than the models presently used by ODNR DW suggests that this added complexity may be beneficial for improving prediction accuracy. The fact that some variables showed up in our models on both logarithmic and original scales suggest that there may be nonlinearities between yellow perch recreational harvest and variables such as yellow perch recreational harvest or effort in the previous years, which if so would explain why our models have better predictive capabilities.

The common issue of overparameterization, such that models with more parameters fit data better, is ameliorated in the current situation because TPE depends on predicting data not used in fitting the model. One remaining concern is the possibility that considering a wide suite of models could be inflating the performance of the selected models when faced with new data that were not included in the selection process. We were not able to find research that was informative on this issue, and simulation testing of selection among all regressions by TPE was beyond the scope of our current efforts. Our best judgment is that these more complex models selected by TPE would provide better predictions than the models currently used by ODNR DW, as long as the combination of predictors remains within the historical range of values present in the data used as part of our evaluation.

The analyses conducted as part of our evaluation accounted for some violations of the assumptions inherent to multiple linear regression (linearity, independence, homoskedasticity); one lingering issue is observation error in stock sizes of yellow perch and walleye populations. These are estimated via stock assessment models, and the predictions from these assessment models clearly can have substantial errors associated with them. Given ODNR DW's focus on prediction, this may not be as large a problem as it would first appear to be. ODNR DW is attempting to derive a predictive equation that will relate yellow perch recreational harvest to a suite of predictor variables. While the existence of substantial observation error would make the estimated relationship a biased characterization of the relationship between harvest and actual stock size, what ODNR DW is interested in here is that the relationship between recreational harvest and historical predicted stock sizes work well for future predictions of (rather than true) stock sizes. The standard problems with measurement error do not apply in this case, but non-constancy of observation error variance, as well as correlated measurement errors could cause some bias in predictions. Our approach, however, of using TPE for model selection means that these factors were taken into account when selecting and evaluating predictive models, and even in the face of such issues the selected models appear to do well at predicting data not used to fit the model. An area to consider in future work is that in practice the projection forward will always use the most recent assessment results of stock size, which are typically estimated most poorly. One possibility would be to actually use in each of the sequential regressions used to calculate TPE the time series of stock sizes that was available when the actual projection of harvest have been done. This was not possible in the current work because we did not have access to the historical stock assessment results. The larger issue to keep in mind is that statistical modeling of complex fishery time series involves assumptions and approximations. It was not possible to evaluate all of these and continual scrutiny of the performance of the adopted approach will be needed.

Acknowledgments

We thank Jim Bence for reviewing a draft of this report. This is technical report T2010-01 of the Quantitative Fisheries Center at Michigan State University.

Table 1. Models with the lowest total prediction error (TPE) for yellow perch recreational harvest by management unit when harvest was estimated directly and indirectly. Total prediction error was calculated for recreational harvest from 2000 to 2006 where models were parameterized with the data from all the years prior to the year for which harvest was predicted (e.g., when predicting 2004 recreational harvest, models were parameterized using 1991-2003 data). The regressor variables from which recreational harvest (or effort) was predicted are shown for each model. When estimating harvest indirectly, predicted recreational effort was multiplied by mean CPUE and fish weight averaged over the previous 3 years.

Area	Model	TPE
	Estimate Harvest Directly	
MU1	LN(MU1_YP_Harvest) = *ƒ*[MU1_YP_EffortPre, MU1_WAE_EffortPre, MU1_YP_Pop, WAE_Pop, LN(MU1_YP_EffortPre), LN(MU1_WAE_EffortPre), LN(MU1_YP_Pop), LN(MU1_YP_HarvestPre)]	0.160
MU2	LN(MU2_YP_Harvest) = *ƒ*[MU2_WAE_EffortPre, MU2_YP_HarvestPre, LN(WAE_Pop), LN(MU2_YP_HarvestPre)]	0.430
MU3	LN(MU3_YP_Harvest = *ƒ*[MU3_WAE_EffortPre, WAE_Pop, LN(MU3_WAE_EffortPre), LN(MU3_YP_MnWeight)]	0.400
	Estimate Harvest Indirectly	
MU1	LN(MU1_YP_Effort) = *ƒ*[MU1_WAE_EffortPre, LN(MU1_YP_EffortPre), LN(MU1_WAE_EffortPre), LN(MU1_YP_Pop), LN(WAE_Pop)]	0.227
MU2	LN(MU2_YP_Effort) = *ƒ*[MU2_WAE_EffortPre, MU2_YP_Pop, LN(MU2_YP_EffortPre), LN(MU2_WAE_EffortPre), LN(MU2_YP_Pop), LN(MU2_YP_HarvestPre)]	0.589
MU3	LN(MU3_YP_Effort) = *ƒ*[MU3_WAE_EffortPre, WAE_Pop, LN(MU3_WAE_EffortPre), LN(MU3_YP_HarvestPre)]	0.398

Table 2. Total prediction error for the models presently used by ODNR DW for predicting yellow perch recreational harvest. Total prediction error was calculated for recreational harvest from 2000 to 2006 where models were parameterized with the data from all the years prior to the year for which harvest was predicted (e.g., when predicted 2004 harvest models were parameterized using 1991-2003 data). The regressor variables from which recreational effort was predicted are shown for each model. When estimating harvest indirectly, predicted recreational effort was multiplied by mean CPUE and fish weight averaged over the previous 3 years.

Area	Model	TPE
MU1	MU1_YP_Effort = f[LN(MU1_YP_Pop), LN(WAE_Pop)]	0.347
MU2	LN(MU2_YP_Effort) = f[MU2_YP_Pop, LN(WAE_Pop)]	1.168
MU3	LN(MU3_YP_Effort) = f[MU3_YP_Pop, LN(WAE_Pop)]	2.395

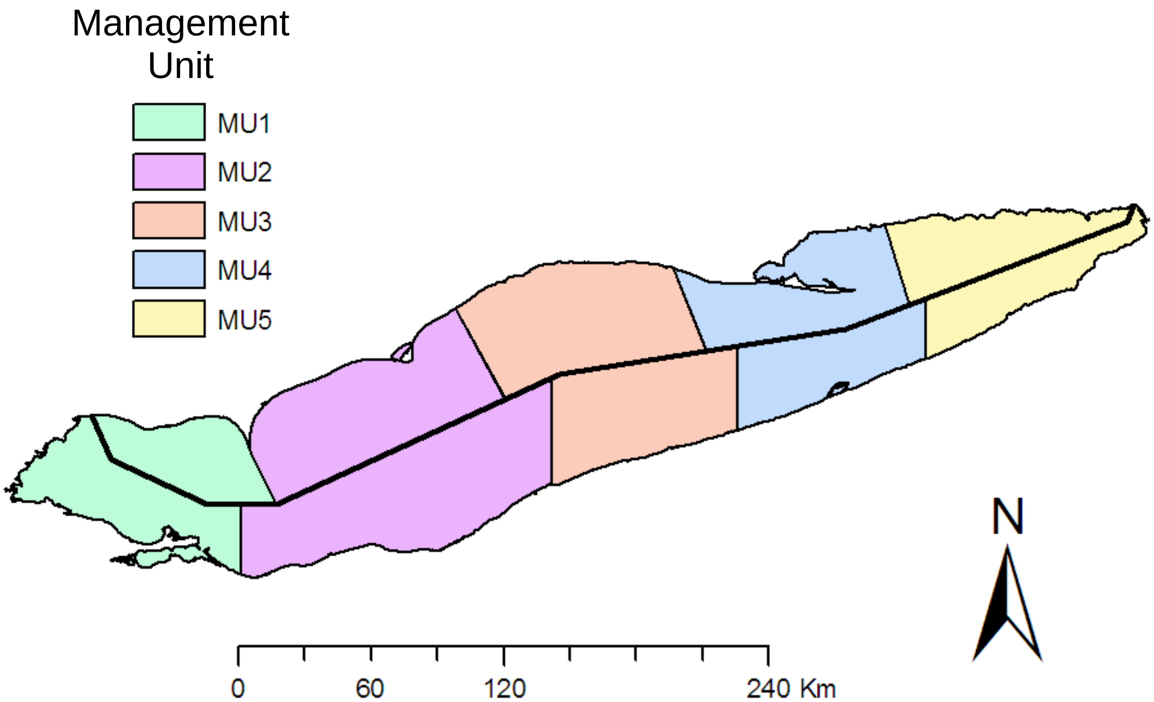

Figure 1. Map of Lake Erie showing the management units (MU) of the lake as well the Canada-US international border.

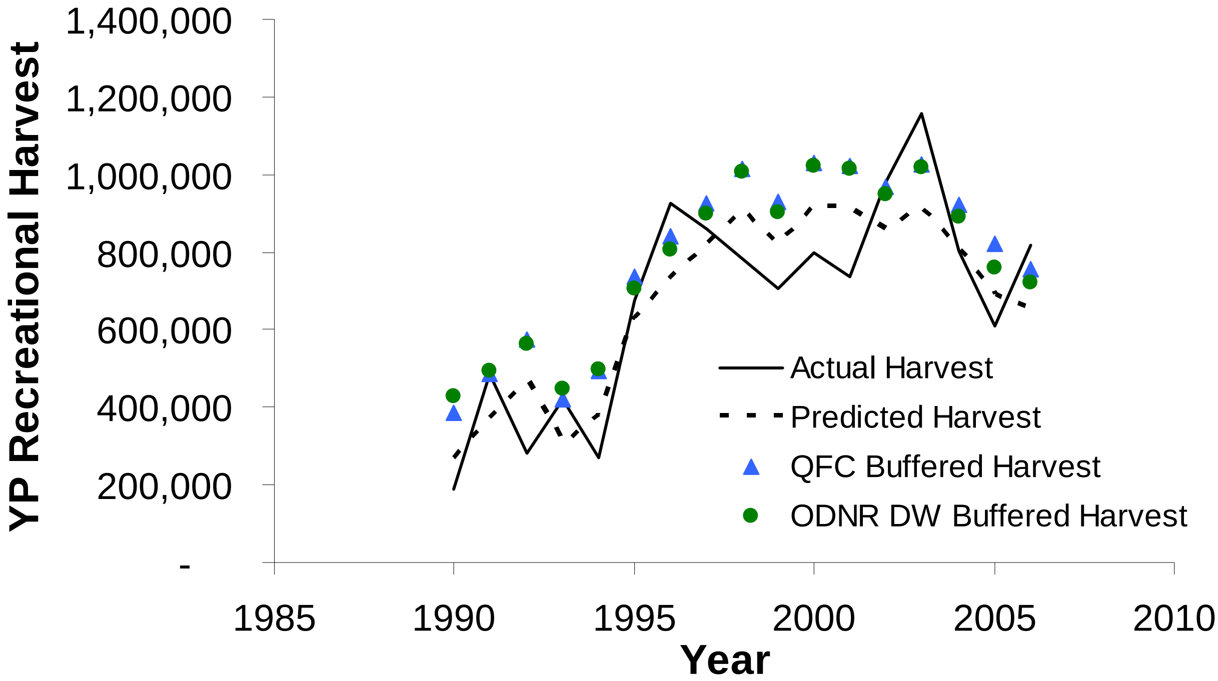

Figure 2. Comparison of buffered yellow perch recreational harvest calculated using the ODNR DW approach described in the answer to Question 3 (ODNR DW Buffered Harvest) and using the general formula for confidence intervals (QFC Buffered Harvest). This comparison was based on the ODNR DW model for direct estimation of yellow perch recreational harvest for MU1. QFC buffered harvest was calculated from an upper 0.625% confidence limit.