A Review of Stock Assessment Methods for Lake Trout and Lake Whitefish in 1836 Treaty Waters of Lake Huron, Lake Michigan and Lake Superior

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1 Introduction

Stock assessments for lake trout and lake whitefish in 1836 treaty waters of Lake Huron, Lake Michigan and Lake Superior have been conducted annually using statistical catch-at-age models since 2001 (although not all areas have been assessed in each year). Statistical catch-at-age has its origins in virtual population analysis (e.g., Pope, 1972) where population statistics such as mortality rates are calculated by tracking cohorts through time in a fishery. This approach is expanded on in statistical catch-at-age by rejecting the assumption that the data used to produce the model are error-free, and instead assuming each datum to be a sample from an underlying statistical distribution (Fournier and Archibald 1982).

While each 1836 treaty waters lake trout and lake whitefish stock assessment area employs a statistical catch-at-age model based on an original design used in 2001, there have been changes to the embedded submodels. The original approach was outlined by Caroffino and Lenart (2011), but there has been no detailed written review of changes to the model structures since 2001. The purpose of this report is to describe the features of each of the 1836 treaty water lake trout and lake whitefish assessment models, focusing on when those features differ from those outlined by Caroffino and Lenart (2011).

A general outline for the original model design, based on the Caroffino and Lenart report (2011), is presented below in section 1.1. Section 2 documents differences between the original design and current lake trout models and section 3 documents differences between the original design and current lake whitefish models. Sections 2 and 3 are divided into subsections that outline the assessment submodels. All changes were identified by reviewing the AD Model Builder code (as of October, 2014) as well as annual reports of the Technical Fisheries Committee for 1836 Treaty Waters. We recognize that these models will continue to evolve, but many features are likely to be retained, and this report will help those using the models now and in the future interpret specific aspects of the models, and better understand what has changed since the start of the 2015 assessment cycle.

1.1 A review of the original model design

The catch-at-age models project population numbers forward through time. The projection is implemented assuming exponential survival as

$$N_{y+1,a+1} = N_{y,a} e^{-Z_{y,a}} \tag{1.1}$$

where $N_{y,a}$ is the number of individuals in age class a in year y and $Z_{y,a}$ is the total instantaneous mortality impacting age class a in year y. Z is the sum of all instantaneous mortality rates

$$Z_{y,a} = F_{y,a} + M_{y,a}^{L} + M^{B} \tag{1.2}$$

including fishing mortality (F), lamprey natural mortality (M^L) and background natural mortality (M^B). The background component to natural mortality is not assigned to a particular source, but data on sea lamprey injuries on fish are used to inform the lamprey-induced mortality component M^L. The sea lamprey mortality rates were estimated externally to the stock assessment model so that process is not described here. Background natural mortality was estimated within the model, but given a prior derived from Pauly's generalized equation (Pauly 1980) that uses growth parameters from fitted von Bertalanffy growth models for populations as well as annual mean water temperature. The mean of the normal prior distribution for the log of background natural mortality was

$$\ln(\widehat{M}) = -0.0238 - 0.277L_{\infty} + 0.655\ln(K) + 0.465\ln(T) \tag{1.3}$$

where K and L_{∞} are von Bertalanffy growth parameter estimates and T is temperature in °C. The standard deviation for the prior distribution was fixed at a single value for all lake trout models based on an analysis of natural mortality rates estimated for lake trout populations (Caroffino and Lenart 2011). Background natural mortality was constant over time.

Annual fishing mortalities-at-age, $F_{y,a}$, represent the rates imposed on fish of a particular age class each year. This rate was some fraction of the maximum annual fishing mortality that acts on fully selected fish, F_y^* (fishing intensity), which is estimated in the model. The fraction was expressed through annual selectivity-at-age, $S_{y,a}$

$$F_{y,a} = S_{y,a}F_y^* \tag{1.4}$$

The original design modeled selectivity as a double logistic curve that can produce a dome-shaped relationship between selectivity and age. This shape is appropriate for gear such as gill nets and trap nets because each can be less effective at catching both small and large individuals. The four parameter double logistic model is the product of one increasing and one decreasing logistic function, and can be expressed

$$S_{y,a}^* = \left[\frac{1}{1+\exp(-\beta_1 a - \beta_{2,y})}\right]\left[1 - \left(\frac{1}{1+\exp(-\beta_3 a - \beta_4)}\right)\right] \tag{1.5}$$

where the β_1 and β_3 parameters represent the slope of the increasing and decreasing logistic functions, respectively, and the $\beta_{2,y}$ and β_4 represent the position of the inflection point of the increasing and decreasing functions. For lake trout, the raw selectivities S^* were then standardized using a reference value $S_{y,r}^*$

$$S_{y,a} = \frac{S_{y,a}^*}{S_{y,r}^*} \tag{1.6}$$

so that the selectivity was set relative to the reference age selectivity. Lake whitefish models differed slightly in that the reference age did not vary by year but instead were always referenced to the first year, so Eqn. 1.6 becomes

$$S_{y,a} = \frac{S_{y,a}^*}{S_{y_1,r}^*} \tag{1.7}$$

Selectivity could change by year; the changes were modeled as a function of the $\beta_{2,y}$ parameter so the change only affects the increasing part of the function. The β_2 parameter depended on year according to a quadratic function

$$\hat{\beta}_{2,y} = \omega_0 + \omega_1(y - y_1) + \omega_2(y - y_1)^2 \tag{1.8}$$

where ω_0, ω_1 and ω_2 are estimated parameters, y is year and y_1 is the first year in the model. If necessary the ω_2 term could be removed (or equivalently fixed to zero and not estimated) making β_2 a simple linear function of year, or β_3 and β_4 in Eqn. 1.5 can be fixed to values and not estimated so that there is no decreasing component to the double logistic.

For simplicity the above formulas do not include a gear type subscript; however, selectivity was modeled by Eqns. 1.5-1.8 separately for surveys as well as commercial and recreational fisheries.

Fishing intensity F^* (fully selected fishing mortality) for a particular gear type was assumed proportional to fishing effort E for that gear type (up to a multiplicative error) such that

$$F_y^* = q\zeta_y E_y \tag{1.9}$$

where the catchability, q (unique to each gear type), represents the baseline proportional relationship, and the "error," ζ_y, is the deviation from strict proportionality (the log of which was assumed to come from a normal distribution).

Recruitment in lake trout models depended on whether the population was supported by wild or hatchery production. Wild recruitment for lake trout models, used in the Lake Superior assessments, was estimated using a random walk

$$N_{a_0,y} = \begin{cases} e^{\ln(N_{y_1,a_0})} & \text{if } y = y_1 \\ e^{\ln(N_{y-1,a_0})+\psi_y} & \text{if } y > y_1 \end{cases} \tag{1.10}$$

where $\ln(N_{y_1,a_0})$was the estimated log recruitment in the first year and ψ is a vector of deviations that describes how much log recruitment changed each year from the amount in the previous year. These log scale deviations were assumed to be normally distributed. Using a random walk assumes correlation in year-to-year recruitment because the most likely value for ψ is zero (Caroffino and Lenart 2011).

Hatchery-supported populations in Lakes Huron and Michigan used a combination of annual total hatchery production and a movement matrix that determined the proportions of fingerlings and yearlings that recruit to each area. These together determined the expected area-specific numbers of fingerlings produced in year $y-1$ (S_{y-1}^f) and yearlings produced in year y (S_y^a). Total estimated annual recruitment R_y also depended on the fingerling-to-yearling conversion ratio κ and a log-scale annual survival parameter φ_y. The equation was

$$R_y = \left(\kappa S_{y-1}^f + S_y^a\right)e^{\varphi_y} \tag{1.11}$$

where values of φ_y that deviated from zero were penalized in the likelihood function. For consistency with Caroffino and Lenart (2011) the mortality term φ_y was included in Eqn. 1.11, however later in this report this age-1 mortality term can be found in the natural mortality sections.

Recruitment in some lake whitefish models was usually estimated on an annual basis, though more complicated methods were also used in some cases (Ebener et al. 2005). During estimation of annual recruitment departures from predictions based on a Ricker stock-recruit function penalized the recruitment estimates (Ebener et al. 2005). This would tend to bring estimates closer to values from the Ricker function, especially when there was not much information in the data suggesting a departure from the Ricker function value. The Ricker model is

$$R_y^* = aG_y e^{-bG_y} \tag{1.12}$$

Where G_y is an estimate of annual egg production, the product of fecundity per unit biomass by age each year ($f_{a,y}$; an external estimate) and the number of spawners $P_{a,y}$

$$G = \sum f_{a,y} P_{a,y} \tag{1.13}$$

The number of spawners is found using the exponential survival equation and an estimate of spawning time

$$P_{a,y} = p^F N_{y,a} e^{-Z\tau} \tag{1.14}$$

where p^F is the proportion of females in the stock and τ is the fraction of one year that has passed before spawning occurs. Both p^F and τ are estimated externally. The differences between the model estimated recruitment each year ($N_{a_0,y}$) and the Ricker estimates (R_y^*) were included in the likelihood function.

The stock assessment model fitting process includes comparing estimates for observed values to those predicted within the model. Fishery catch-at-age and (for lake trout) survey CPUE-at-age are used to fit the model. Catch-at-age for each fishery is estimated using Baranov's catch equation

$$C_{a,y} = \frac{F_{a,y}}{Z_{a,y}} N_{a,y} (1 - e^{-Z_{a,y}}) \tag{1.15}$$

where $C_{a,y}$ is the catch-at-age a in year y. This equation is used for the various gear types in the commercial fisheries as well as for any recreational fisheries. Bycatch and under-reporting are accounted for by adding external estimates to the directly observed catch (summed over ages) outside the model so that all known sources of fish death due to fishing were accounted for in the observed catch values used when fitting the model. The models treated the total catch each year and the proportions of catch-at-age for each year as separate data sources.

The other observational data (in lake trout models only) come from fishery-independent surveys. In the stock assessment models, the predicted catch-per-unit-effort is:

$$CPUE_{y,a} = qS_{y,a} N_{y^I,a} \tag{1.16}$$

where $CPUE_{y,a}$ is the year- and age-specific catch-per-unit-effort, S and q are survey-specific selectivities and catchabilities, and $N_{y^I,a}$ is the abundance of age class a adjusted for the time of year of the survey. The estimation of numbers at the time of the survey assumes exponential survival at a constant annual mortality rate

$$N_{y^I,a} = N_{y,a} e^{-\tau_I (F+M)} \tag{1.17}$$

where τ_I is the survey timing as a proportion of the year. As with fishery catch, survey CPUE was treated as two components, one the total and the other the proportions at age each year, when comparing model predictions with data.

While the deviations between observed and predicted total CPUE could arise from either temporal changes in catchability or measurement error, q was viewed as constant over time. Thus the fit to the total survey CPUE was weighted by annual estimates of standard deviations, which reflected estimated measurement error. Survey catch-per-unit-effort (for lake trout; no surveys were included in whitefish assessment models) was standardized prior to inclusion in the stock assessment model using a statistical model, but that process occurred outside the assessments and so is not discussed here.

The parameters in the models were adjusted during fitting so that they minimized an objective function. The objective function was:

$$-L = \sum_{i=1}^{k} -\lambda_i L_i \tag{1.18}$$

where k is the number of components in the likelihood function, λ_i is the weighting of component i, and L_i was typically either the log-likelihood of data component i given specific parameter values, or log prior densities for parameters (or groups of parameters assumed to share a common distribution). Priors for most parameters were diffuse or bounded uniforms and were not included in Eqn. 1.18. In most cases the weights were set to 1.0 (so actual log likelihood or log prior densities were used) and other values were used only to test the sensitivity of the model to a particular component of the objective function (Caroffino and Lenart 2011). Different error distributions can be assumed for each of the likelihood components. This approach to estimation (fixing variances or ratios of variances and using point estimates obtained by minimizing 1.18) is variously known as "error in variables", "penalized likelihood", or "highest posterior density estimation" (Schnute 1994; Wilberg et al. 2010). Although it has been referred to as maximum likelihood both in MSC documents and in the broader literature, it differs from maximum likelihood because some components of the objective function do not involve the likelihood of data but instead departures of parameters from prior expectations.

2 Lake trout assessment model summaries

A description of the submodels for each lake trout assessment area as they stood at the end of the 2014 assessment cycle including any important differences between these models and the approach described by Caroffino and Lenart (2011) is given below.

2.1 Selectivity

MI-5 selectivity

MI-5 uses a gamma function to describe commercial, recreational, large mesh survey and graded mesh survey selectivities. The full gamma probability density function can be parameterized in terms of rate and shape as

$$g(a;\alpha,\beta) = \frac{\beta^{\alpha+1}}{\Gamma(\alpha+1)} a^{\alpha} e^{-\beta a} \tag{2.1.1}$$

where α and β are parameters and a is the data (ages here). The term $\frac{\beta^{\alpha+1}}{\Gamma(\alpha+1)}$ serves to scale the response, but since the selectivity is later standardized this term can be removed. Thus the function coded in the assessment model, including the capacity to vary the β parameter by year, becomes

$$g(a;\alpha,\beta,y) = a^{\alpha} e^{-\beta_y a} \tag{2.1.2}$$

Selectivity for these two indices varies over time via a random walk of the β parameter. The random walk is implemented as

$$\beta_y = \begin{cases} e^{k_{y_1}} & \text{if } y = y_1 \\ e^{\log(\beta_{y-1})+k_y} & \text{if } y > y_1 \end{cases} \tag{2.1.3}$$

where k_{y_1} is freely estimated and a log-scale deviation k is estimated for each year. The first and last random walk years are given in the table below. The log scale deviations were assumed to be normally distributed in the likelihood function. Time-varying selectivity is not used for the recreational fishery, and the recreational selectivity parameter β that applies across all years is estimated as a free parameter.

For the commercial and recreational fisheries and the large mesh survey, the selectivity for the plus group is set equal to that of the next-to-last age.

Selectivity is standardized by dividing each estimated selectivity by the max annual selectivity

$$S_{a,y} = \frac{g(a;\alpha,\beta,y)}{\max[g(A;\alpha,\beta,y)]} \tag{2.1.4}$$

where A represents the vector of all modeled ages. Thus selectivity reaches a peak value each year of 1.0.

Commercial selectivity was initially estimated for 1986, then varied over time starting from that year according to a random walk (Eqn. 2.1.3) that continued through 1999. Selectivity for 2001, 2002, and 2005 was assumed equal to the 1999 selectivity. Single values for the selectivity parameters α and β were estimated and applied to the years 2000, 2003, 2004, and 2006-2013 (the last year). Thus for the years after the random walk was used, there were two different selectivity vectors applied to two different non-contiguous blocks of years.

Prior to 1986 commercial selectivities were assumed equal to selectivity from the large mesh survey in corresponding years. Selectivities before the first year of recreational fishing age-composition data were set equal to the 1988 estimates. Finally, selectivity in the recreational fishery, large mesh survey and graded mesh survey for the last two years are set equal to selectivity in the third-to-last year (2010).

Selectivity information

Description	Model Years
Large mesh selectivity random walk	1976-2011
Commercial fishery selectivity random walk	1987-1999
Commercial fishery selectivity time blocks	2000, 2003-2004, 2006-2013, 2001-2002, 2005

MI-6 selectivity

MI-6 also uses a gamma function to describe selectivity (Eq. 2.1.2). The time-varying component is a random walk as in MI-5 (2.1.3), and is employed for the commercial fishery as well as for the large mesh survey. Active random walk parameters are included in the likelihood function and assumed to be normally distributed. The graded mesh survey has no time-varying component and (as in MI-5) only data (and thus selectivity estimates) for ages 4 and 5 are used. Standardization is achieved using the max function (Eqn. 2.1.4). The difference in the use of Eqns. 2.1.2-2.1.4 between MI-6 and MI-5 is that in MI-6 recreational fishing has time-varying selectivity, whereas this was constant in MI-5. The initial years for the random walks (β_{y_1}) were 1978 for both the large mesh and commercial fishery, and the random walks continue through the third-to-last year of the model (2010) in both cases. Selectivities in the last two model years are set equal to the 2010 selectivity. Selectivity in the last two age classes for the two fisheries and two surveys are also assumed equal.

Recreational selectivity is constant from 1987-2000 and from 2006-2013, and estimated using a gamma function. During 2001-2005 MI-6 has some particular adjustments related to changes in size regulations for the recreational fishery. Recreational selectivities from 2001-2005 are calculated using a fixed selectivity estimate multiplied by an adjustment vector. The adjustments occur after the initial baseline selectivity that is used in the other years has been estimated (i.e., using the random walk/gamma distribution). The adjustments are supplied in the ADMB data file. The (non-standardized) selectivity S^* for years 2001-2005 is

$$S_a^* = g(a; \alpha, \beta, y^*) J_{a,b} \qquad (2.1.5)$$

where $g(a; \alpha, \beta, y^*)$ is the raw selectivity at age a for year y^* calculated using the gamma function (Eqn. 2.1.2) and J is the externally supplied vector of recreational fishery selectivity adjustments at age a for adjustment block b. y^* is used instead of y because in many cases the selectivity that is employed for a given year uses a different year's selectivity for the calculation (see table below). Recreational selectivity during 2006-2013 is assumed equal to the 2000 selectivity. The result of these adjustments is that the recreational selectivities estimated using the random walk/gamma function after the year 2000 are overwritten in the selectivity matrix (though this does not affect the objective function because the recreational fishery random walk parameters were excluded).

Time block selectivity adjustments for MI-6. y^* and b are from Eqn. 2.1.5.

Selectivity time block adjustments

Model year	y^*	b
2001	2000	1
2002	2001	2
2003-2005	2002	3

Random walk information

Index	First Year	Last Year
Commercial Fishery	1978	2013
Large Mesh Survey	1978	2013
Recreational fishery (1)	1988	2005
Recreational fishery (2)	2007	2011

MI-7 (2012) selectivity

The approach to selectivity in the MI-7 area is the same as in MI-5 and MI-6, with some minor variations. Selectivity varies over time via a random walk (Eqn. 2.1.3) in the recreational and commercial fisheries and the large mesh survey but not the graded mesh survey. As in the other areas, the graded mesh survey uses only data for ages for ages 4 and 5 and selectivity is estimated only for those ages using a gamma function. Graded mesh selectivity does not vary over time. Plus group selectivity for the recreational and commercial fisheries as well as for the large mesh survey is assumed equal to selectivity in the next-to-last age group.

Commercial selectivity is not estimated before 1985, and for these years the model borrows the large mesh survey selectivity and applies it to the commercial fishery. Selectivity for the two fisheries and the large mesh survey were assumed constant during the final three years of the model.

Random walk information

Index	First Year	Last Year
Recreational Fishery	1985	2009
Commercial Fishery	1976	2009
Large Mesh Survey	1976	2009

MM-123 selectivity

The MM-123 stock area uses a double logistic function to estimate selectivity-at-age. Recreational fishery selectivity is divided into three time blocks while commercial and survey selectivity are constant through time.

The function used to describe (non-standardized) selectivity at age a ($S^*_{a,b}$) in MM-123 is

$$S^*_{a,b} = \left[\frac{1}{1+\exp(-\beta_1 a - \beta_{2,b})}\right]\left[1-\left(\frac{1}{1+\exp(-\beta_3 a - \beta_4)}\right)\right] \tag{2.1.6}$$

which is similar to Eqn. 1.5, except the S and β subscripts reference time blocks rather than annual estimates. Selectivity in the commercial fishery and the survey do not vary through time so these parameters are static. The recreational fishery, however, is divided into three time-blocks, and the $\beta_{2,b}$ parameter indicates which time block should be used. Each time blocks' β_2 parameters are freely estimated. The first time block is up to and including 1992, the second runs from 1993-2011 and the third from 2012-2013.

The raw selectivities are standardized using the max function

$$S_{a,b} = \frac{S^*_{a,b}}{\max\left(S^*_{a,b}\right)} \tag{2.1.7}$$

so selectivity peaks at 1.0 for the fully selected age each year.

Selectivity information

Description	Model Years
Recreational fishery time blocks	1985-1991, 1992-2011, 2012-2013
Commercial fishery random walk	1982-2013

MM-4 selectivity

The MM-4 stock area uses double logistic models to estimate selectivity-at-age (Eqn. 1.5). Survey selectivity varies over time according to a random walk where the β_2 parameter changes on an annual basis, governing the position of the inflection of the increasing part of the double logistic curve. The random walk is implemented in the same manner as in Lake Superior (Eqn. 2.1.3) but is used in a double logistic instead of a gamma function.

As in MM-123, the β_2 (Eqn. 2.1.6) parameter for the recreational fishery is divided into four time blocks and so does not vary each year. The β_2 parameter for each of these time blocks is freely estimated, so there is no prior expectation that the parameter value estimated for a time block will be similar to that of the previous time block (as for adjacent years in the random walk approach). The time blocks are: (1) prior to 1992, (2) 1992-1996, (3) 1997-2010, and (4) 2011-2013. After the four initial blocks are developed, two additional time blocks are created for 2003-2005 and 2006-2010 by adjusting the baseline estimated selectivity for the 1997-2010 block by a fixed (externally calculated) adjustment for each of the two new time blocks, following the same adjustment procedure used for the Lake Superior stock MI-6 (Eqn. 2.1.5). This is accomplished by multiplying the 2002 recreational fishery selectivity by an adjustment to produce the selectivity for 2003-2005. During 2006-2010 the selectivity is the product of an additional adjustment and the 2005 recreational selectivity.

Selectivity information

Description	Model Years
Recreational fishery selectivity time blocks	1985-1991, 1992-1996, 1997-2002, 2003-2005, 2006-2010, 2011-2013
Survey selectivity random walk	1982-2013

MM-5 selectivity

The model for the MM-5 stock area uses double logistic functions for selectivity-at-age (Eqn. 1.5). Survey selectivity varies over time according to a random walk (Eqn. 2.1.3), while recreational fishery selectivity varies in five time blocks. Commercial selectivity is time-invariant in MM-5 and each of the parameter values are fixed outside the model. Survey selectivity varies over time using a random walk that is initiated in 1981, the first model year, but that breaks during 1990 to 1996. During this time block (b), the parameter modeled by a random walk in years outside this block is set at

$$\beta_{2,b} = e^{\log(\beta_{2,1989})+k_{1989}} \tag{2.1.8}$$

This is not equivalent to Eqn. 2.1.3; there the random walk parameter (from the gamma function in that case) is used with the random walk deviation for the year $y+1$ rather than the deviation in year y, as is used here. After 1996 the random walk is resumed, initialized with the value used for 1989-1996. The final year is 2010. The random walk deviates (k in Eqn. 2.1.3) are included in the likelihood and assumed to follow a normal distribution. These are coded as separate likelihood components for the period before and after the 1990-1996 time block in the objective function, with different standard deviations.

Recreational selectivity is initially divided into three time-blocks where the inflection point of the first logistic model (β_2 in Eqn. 2.1.6) is freely estimated. The time-blocks are: (1) prior to 2006, (2) during 2006 to 2011, and (3) after 2011. Selectivity is in some cases further altered, as in MM-123 and MM-4, by multiplying the estimated selectivity by a fixed adjustment (as in Eqn. 2.1.5). During 2001-2002 selectivity is the product of the 2000 selectivity and the selectivity adjustment vector. During 2003-2005 selectivity is the product of the 2002 selectivity and the adjustment vector. In all there are five recreational selectivity blocks.

Selectivity information

Description	Model Years
Recreational fishery selectivity time blocks	1985-2000, 2001-2002, 2003-2005, 2006-2011, 2012-2013
Survey selectivity random walk	1982-1989, 1997-2010

MM-67 (2012) selectivity

Selectivity in MM 6-7 uses a double logistic function. The commercial and survey selectivities all vary over time by changing the β_2 parameter (Eqn. 1.5). This parameter varies according to a quadratic function as presented in the original model design (Eqn. 1.8). These affect the offset parameter ω_0 in Eqn. 1.8 such that

$$\omega_{0,b} = e^{\varphi_b} \tag{2.1.9}$$

where φ_b is the log-scale estimate for ω during time block b. The recreational fishery estimates different sets of all four double logistic parameters (Eqn. 1.5) during two time blocks (i.e., β_1^{b1}, β_2^{b1}, β_3^{b1} and β_4^{b1} for block one and β_1^{b2}, β_2^{b2}, β_3^{b2} and β_4^{b2} for block 2). The two time blocks run from 1985-2002 and 2003-2011. Selectivity is standardized using the max function (Eqn. 2.1.7).

Selectivity information

Description	Model Years
Recreational fishery selectivity time blocks	1985-2002, 2003-2011
Commercial fishery selectivity quadratic	1982-2011
Survey selectivity quadratic	1982-2011

MH-1 selectivity

Selectivity-at-age in MH-1 assumes a lognormal distribution. In addition, the approach differs from the methods previously described because it estimates selectivity at age using length-at-age. Non-standardized selectivity S^* at age a in year y is

$$S^*_{y,a} = \frac{1}{\sigma_y L_a \sqrt{2\pi}} \exp\left(-\frac{(\ln(L_a) - \mu)^2}{2\sigma_y^2}\right) \tag{2.1.10}$$

where σ_y is the lognormal standard deviation in year y, L_a is the average length at age a and μ is the lognormal mean. Before standardization, the plus age group selectivity is set equal to the second-to-last age group selectivity. The selectivity is standardized by the lognormal pdf at a length equal to μ

$$S_{\mu,y} = \frac{1}{\sigma_y e^{\mu} \sqrt{2\pi}} \tag{2.1.11}$$

The standardized selectivity is

$$S_{y,a} = \frac{S^*_{y,a}}{S_{\mu,y}} \tag{2.1.12}$$

The selectivity function in most years is time-varying with respect to the parameter σ which is freely estimated each year. The exception is recreational fishery selectivity during 1977-1984, which is set to equal the estimated 1985 recreational selectivity. Note that 2.1.10 is maximized at $\exp(\mu - \frac{\sigma^2}{2})$, so selectivity at age can exceed 1.

MH-2 selectivity

Selectivity estimation in the MH-2 area takes the same approach as in MH-1, including the block of recreational fishery selectivity during 1977-1984.

2.2 Catchability

MI-5 catchability

Recreational catchability by year in MI-5, q_y^R, is estimated using a mean parameter and a vector of deviations that are constrained to sum to zero (i.e., white noise), so

$$q_y^R = e^{\ln\bar{q}^R + q_y^{dev}} \tag{2.2.1}$$

where $\ln\bar{q}^R$ is the estimated log scale mean recreational catchability and q_y^{dev} is the log scale annual recreational catchability deviation parameter. The vector of log-scale deviations is included in the likelihood function and assumed to be normally distributed (though this is equivalent to a lognormal assumption; see section 2.9). In 1989 there were no data on recreational fishing effort so catchability was not estimated; fishing mortality in this year was assumed to be the mean of the 1988 and 1990 estimates.

Commercial catchability is estimated in the same manner as recreational catchability from 1986 on (i.e., as white noise). During 1975-1980 fishing mortality was estimated independently for each year, and during 1981-1985 it was fixed at 0.00248 so during those years commercial catchability was not estimated at all. To combat fitting issues with potentially correlated estimates for recreational and commercial catchability, mean commercial catchability, $\ln\bar{q}^C$ is calculated as a deviation from mean recreational catchability, so

$$\ln\bar{q}^C = \ln\bar{q}^R + \delta^C \tag{2.2.2}$$

Where δ^C is the log scale estimated difference between the two mean catchabilities.

Catchability for the large mesh and graded mesh surveys are not assumed to vary over time and those parameters are freely estimated on a log scale.

Catchability information

Description	Model Years
Commercial fishery dev vector	1986-2013
Recreational fishery dev vector	1984-1988, 1990-2013

MI-6 catchability

The approach to commercial and recreational catchabilities for MI-6 is the same as for MI-5. There is again no recreational fishing effort data for 1989 so fishing mortality is estimated to be the mean for the 1988 and 1990 *F* estimates. There is no effort data prior to 1987 for the recreational fishery or prior to 1978 for the commercial fishery, so catchability was not estimated in those years; instead recreational or commercial fishing mortality was set equal to the value in the first year of available effort data for the respective fishery (1987 for the recreational fishery and 1978 for the commercial fishery). The large mesh and graded mesh survey catchabilities in MI-6 do not vary over time, but unlike in MI-5 they are re-parameterized to avoid fitting problems,

$$\ln q^{G} = \ln q^{L} + \delta^{G} \tag{2.2.3}$$

where $\ln q^{G}$ is the log scale catchability from the graded mesh survey, $\ln q^{L}$ the log scale catchability from the large mesh survey and δ^{G} the log scale difference between the two catchabilities. Log-scale large mesh survey catchability (as a single value) is included in the likelihood function and measured relative to a prior, assuming a normal distribution.

Catchability information

Description	Model Years
Recreational fishery random walk	1991-2013
Commercial fishery random walk	1978-2013

MI-7 (2012) catchability

Catchability in MI-7 is estimated in the same manner as in MI-5, but with some modifications. During 1975 (the first model year) to 1979 there were no commercial fishing effort data so *F* was estimated directly, assuming that commercial fishing mortality in 1975 and 1976 was the same. There was also no 1989 recreational effort data, so that year's fishing mortality is the average of the 1988 and 1990 estimates. The recreational and commercial deviates (e.g., q_{y}^{dev} from Eqn. 2.2.1) were included in the objective function and assumed to follow a normal distribution.

Catchability information

Description	Model Years
Commercial fishery dev vector	1980-2011
Recreational fishery dev vector	1984-1988, 1990-2011

MM-123 catchability

Catchabilities for commercial and recreational fisheries in MM-123 vary annually according to a random walk. This is expressed

$$\mathrm{q}_{y} = \begin{cases} e^{\log(q_{y_1})} & if\ y = y_1 \\ e^{\log(q_{y-1}) e^{d_y}} & if\ y > y_1 \end{cases} \tag{2.2.4}$$

where d_y is the log scale catchability deviation in year y and the first year of available commercial fishery data is $y_1 = 1981$. This random walk is not equivalent to other random walks used in 1836 treaty waters (e.g., Eqn. 2.1.3, or most other applications) because the deviations are implemented as products on the log scale rather than sums. The same process is used to estimate recreational catchability, except the first year of available data is $y_1 = 1985$. Commercial and recreational fishing mortalities from 1981-1984 are set equal to the 1985 estimates.

Survey catchability in MM-123 does not vary by year and the single value is freely estimated on a log scale.

Catchability information

Description	Model Years
Recreational fishery random walk	1986-2013
Commercial fishery random walk	1982-2013

MM-4 catchability

MM-4 commercial and recreational catchabilities are estimated in the same way as in MM-123, except the catchability random walk proceeds according to the methods presented earlier. The random walk is

$$\mathrm{q}_y = \begin{cases} e^{\eta_1} & if\ y = y_1 \\ e^{\log(q_{y-1})+\eta_y} & if\ y > y_1 \end{cases} \tag{2.2.5}$$

Where η is the random walk deviation vector and η_1 is the freely estimated year 1 catchability on a log scale. This is the typical implementation of a random walk (e.g., Eqn. 2.1.3).

Survey catchability in MM-123 does not vary by year and the single value is freely estimated on a log scale.

Catchability information

Description	Model Years
Commercial fishery random walk	1982-2013
Recreational fishery random walk	1986-2013

MM-5 catchability

The method to estimate commercial catchability in MM-5 before 1990, between 1993 and 2007 and in 2010 is the same as in MM-123, except the series is not continuous. In 1990-1992 and 2008-2009 catchability is not estimated because there was no commercial gill net fishery. To initiate the random walk in years following a year that was not estimated (i.e., in 1993 and 2010) a baseline catchability is required. The 1993 catchability was estimated using catchability for 1989

$$q^C_{1993} = \exp(\ln q^C_{1989})d_{1993} \tag{2.2.6}$$

and the previous value used for the 2010 estimate was from 2007

$$q^C_{2010} = \exp(\ln q^C_{2007})d_{2010} \tag{2.2.7}$$

The random walk used here is not equivalent to other random walks in the lake trout assessments (e.g., Eqn. 2.2.5) because the random walk deviation parameter (d) is not estimated on a log scale. During the years when there was no catch data, commercial fishing mortality was freely estimated as a single parameter that applied to all such years

$$F_g = e^{lnF_g} \tag{2.2.8}$$

where F_g is the fishing mortality during a year with no commercial data.

The methods for estimating recreational fishery catchability are identical to those used in MM-123. Recreational catchability was not estimated prior to 1985; those values are set equal to the 1985 estimate.

Survey catchability does not vary with time and is estimated as a free parameter on a log scale.

Catchability information

Description	Model Years
Commercial fishery catchability random walk	1982-1989, 1996-2007
Recreational fishery random walk	1982-2013

MM-67 (2012) catchability

No catchability was estimated for the commercial fishery; annual fishing mortality was instead estimated directly on a log scale for all years. Recreational catchability was estimated for all model years as the deviation from a mean using a white noise vector that is constrained to sum to zero, so

$$q_y^R = e^{\ln\bar{q}^R + d_y^R} \tag{2.2.9}$$

where $\ln\bar{q}^R$ is the estimated log scale mean recreational catchability and d_y^R is the log scale annual recreational catchability deviation parameter. Survey catchability was estimated as a single, time-invariant parameter. The white noise vector d is assumed to follow a normal distribution in the likelihood function.

Catchability information

Description	Model Years
Recreational fishery catchability dev vector	1985-2011

MH-1 catchability

Both the commercial and survey catchabilities in the MH-1 area are estimated annually as free parameters on a log scale as

$$q_y = e^{\ln q_y} \tag{2.2.10}$$

where q_y is the catchability in year y and $\ln q_y$ is the log-scale estimated catchability for year y. These catchability parameters are estimated with respect to priors in the likelihood function that are organized into time blocks. The prior time blocks for survey catchability were: 1977-1981, 1982-1996 and 1997-2013. The commercial catchability time blocks were: 1982-1988, 1989-1993 and 1994-2013.

Recreational fishery catchability follows a random walk, and is estimated as in Eqn. 2.2.5. The log-scale random walk deviates are assumed to follow a normal distribution in the likelihood function.

Description	Model Years
Recreational fishery random walk	1986-2013

MH-2 catchability

Catchability in MH-2 is implemented in the same manner as in MH-1.

Description	Model Years
Recreational fishery random walk	1986-2013

2.3 Recruitment

MI-5 Recruitment

Recruitment in any year prior to the last year is a function of two log-scale parameters, an average recruitment ($\bar{r}$) and a vector of annual recruitment deviations (r_y^{dev})

$$R_y = e^{\bar{r}+r_y^{dev}} \tag{2.3.1}$$

where $\bar{r}$ and r_y^{dev} are model estimated parameters. The white noise vector of deviations is constrained to sum to zero, included in the likelihood function and assumed to follow a normal distribution. Recruitment in the last model year is set equal to recruitment in the 2nd to last model year, rather than following Eqn. 2.3.1.

MI-6 and MI-7 (2012) Recruitment

Recruitment in MI-6 follows a random walk. Recruitment in the first model year is an estimated parameter, and in all subsequent years (save the last year) is estimated:

$$R_y = \begin{cases} e^{r_1} & if\ y = y_1 \\ e^{r_{y-1}+r_y^{dev}} & if\ y > y_1 \end{cases} \tag{2.3.2}$$

where r_1 is the log-scale year 1 recruitment, r_{y-1} is recruitment in the previous year and r_y^{dev} is the annual deviation. Recruitment in the last model population year is set equal to that in the second to last year rather than following Eqn. 2.3.2. Log-scale random walk deviations are assumed to be normally distributed in the objective function.

MM-123 and MM-67 (2012) Recruitment

The approach to recruitment is identical to that in MM-4 and MM-5 (see below), except that some individuals recruit to the refuge (see *MM-123 and MM-67 (2012) Refuge*).

MM-4 and MM-5 Recruitment

No wild recruitment is assumed. Recruitment comes from stocked fingerlings and yearlings that have moved into the area. A migration matrix from the stocking sites is a fixed input to the model, along with a fingerling to yearling conversion ratio, as both classes are stocked, so

$$R_y = \kappa S_{y-1}^f + S_y^a \tag{2.3.3}$$

where R_y is the total recruits, κ is the conversion ratio, and S_{y-1}^f and S_y^a are the number of fingerlings stocked in the previous year or yearlings stocked in the current year that were assumed to have moved from the stocking site to the MM-4 area.

MH-1 and MH-2 Recruitment

Recruitment in MH-1 is the sum of wild recruitment and stocked recruitment. Stocked recruitment in year y is estimated in the same way as in MM-4 and MM-5 (Eqn. 2.3.3). Wild recruitment in year y (R_y^W) is estimated as a free parameter on a log scale

$$R_y^W = e^{r_y} \tag{2.3.4}$$

and total recruitment R_y is

$$R_y = R_y^H + R_y^W \tag{2.3.5}$$

where R_y^H is stocked recruitment (R_y from eqn. 2.3.3). Wild recruits are age-4 fish, while stocked fish enter the model at age-1.

2.4 Numbers-at-age

MI-5 Numbers-at-age

The general approach to numbers-at-age in MI-5 is consistent with the original 1998 model design, but there are some modifications for deriving the initial population estimates.

Age classes 2-7 in the initial model year are estimated as model parameters

$$I_a = e^{i_a} \tag{2.4.1}$$

where I is the initial estimated number of fish in age class a and i is the log-scale parameter estimate. Age classes 8-15 are estimated assuming exponential survival from the age-7 estimate using the age-9 total mortality estimate for the first model year $z_{1,9}$

$$N_{a+1} = N_a e^{-e^{z_{1,9}}} \tag{2.4.2}$$

where a ranges from 7 to 14, the second to last model age. Eqn. 2.4.2 is the way the code is written; however, the intent is likely for the right side to be $N_a e^{-z_{1,9}}$.

MI-6 Numbers-at-age

MI-6 differs from MI-5 in the estimation of initial numbers-at-age. Ages 2-10 are estimated as independent log-scale parameters as in Eqn. 2.4.1. However, numbers in each of the last five age classes in the first year are identical, and set equal to the number of individuals in the 10th age class (i.e., age classes 10-15 all have the same number of fish).

MI-7 (2012) Numbers-at-age

The MI-7 initial numbers-at-age are estimated in the same manner as in MI-6; however, the last five age classes have equal numbers of individuals instead of the last six age classes.

MM123, MM4, MM5 and MM-67 (2012) Numbers-at-age

The initial numbers-at-age for particular age classes in Lake Michigan are estimated as in Eqn. 2.4.1. The particulars of which age classes are estimated depend on the area. In area MM-5 ages 2-11 are estimated and ages 12-15 are set to zero and in MM-123, MM-4 and MM-67 ages 2-12 are estimated and 13-15 are set to zero.

MH-1 and MH-2 Numbers-at-age

Numbers-at-age in all areas follow the general catch-at-age population dynamic process given above (Eqn. 1.17) that assumes exponential survival. These two areas are slightly different however because they track wild and hatchery numbers-at-age separately, with the initial population estimates carried through with the hatchery group. The first five ages classes (area MH-1) and ten age classes (area MH-2) in model year y_1 for the hatchery-raised group are estimated as

$$N^H_{y_1,a} = e^{i_a} \tag{2.4.3}$$

where i_a represents the logged first model year's number of fish in age class a. Age classes 6-15 (area MH-1) and 11-15 (MH-2) are set to zero. The first model year in MH-1 is 1977 and in MH-2 is 1984. The population dynamic equations are

$$N^H_{y+1,a+1} = N^H_{y,a} e^{-F_{y,a}-M} \tag{2.4.4}$$

and

$$N^W_{y+1,a+1} = N^W_{y,a} e^{-F_{y,a}-M} \tag{2.4.5}$$

where the H superscript indicates hatchery-raised individuals and W the wild individuals. These are summed to estimate the total population numbers-at-age

$$N_{y,a} = N^W_{y,a} + N^H_{y,a} \tag{2.4.6}$$

where $N_{y,a}$ is the total number by year and age. Hatchery fish enter the model at age-1 while wild recruitment occurs at age-4 in both areas.

Further, the plus groups are dealt with in a unique manner. The model assumes the numbers in the plus group to be the result of an infinite geometric series e.g.,

$$1 + x + x^2 + x^3 \ldots = \frac{1}{1 - x} \tag{2.4.7}$$

which is then multiplied by the numbers alive in the age before the plus group in the previous year. The numbers in the plus group can be written

$$N_{y+1,p} = N_{y,p}(e^{-Z} + e^{-2Z} + e^{-3Z} \ldots) \tag{2.4.8}$$

and, using 2.4.7, this becomes

$$N_{y+1,p} = N_{y,p}\left(\frac{1}{1 - e^{-Z_{y+1,p}}} - 1\right) \tag{2.4.9}$$

Note that this treatment assumes the plus group is the number expected if numbers reaching the plus age in every previous year and subsequent mortality up to the year the plus group is being calculated for remained constant and equaled the values for the group first reaching the plus group in the current year. If these assumptions were actually correct, the plus group would not be changing over time.

2.5 Catch-at-age

MI-5 MI-6 and MI-7 (2012) Catch-at-age

Predicted catch-at-age for the commercial and recreational fisheries uses Baranov's catch equation (Eqn. 1.15). Lake Superior areas also incorporate ageing error when converting predictions of actual catch-at-age to predictions of observed catch-at-age. This is accomplished using an age error matrix that determines the proportion of individuals of a given estimated age that belong to each of the true age classes. The calculation is

$$C^*_{y,a} = AE_a C_{y,a} \tag{2.5.1}$$

where AE is the ageing error matrix, $C^*_{y,a}$is the final catch-at-age estimate for age class a in year y to be compared with the data and $C_{y,a}$ is the model-predicted catch-at-age (Eqn. 1.15). For these areas two ageing error matrices are used, the first for the years 1975-1988 and the second for the years 1989-2013.

MM123, MM-4, MM-5 and MM-67 (2012) Catch-at-age

No adjustments to the estimation of catch-at-age are made for these areas.

MH-1 and MH-2 Catch-at-age

Catch-at-age in these areas is calculated as specified in section 1, except that wild catch and stocked catch are determined separately and then combined following the method for estimating numbers-at-age for stocked catch

$$C^H_{a,y} = \frac{F_{a,y}}{Z_{a,y}} N^H_{a,y}(1 - e^{-F_{a,y} - M_{a,y}}) \tag{2.5.2}$$

and wild catch

$$C_{a,y}^{W} = \frac{F_{a,y}}{Z_{a,y}} N_{a,y}^{W} (1 - e^{-F_{a,y} - M_{a,y}}) \tag{2.5.3}$$

so total catch is

$$C_{a,y} = C_{a,y}^{H} + C_{a,y}^{W} \tag{2.5.4}$$

where $C_{a,y}$ is the total catch.

2.6 Natural mortality

MI-5, MI-6, MI-7 (2012) Natural mortality

Lake Superior natural mortality in year y for age a is comprised of lamprey mortality and background mortality

$$M_{y,a} = M^{B} + M_{y,a}^{L} \tag{2.6.1}$$

where lamprey mortality is M^{L} and background mortality is M^{B}. Background mortality is estimated within the model on a log scale and lamprey mortality by age and year is a specified input to the model. Log-scale natural mortality is fit in the likelihood function as a deviation from a prior that is specified externally using Pauly's equation (Eqn. 1.3).

MM-123, MM-4, MM-5 and MM-67 (2012) natural mortality

As in Lake Superior, natural mortality is the sum of background mortality and lamprey mortality. Unlike in Lake Superior, there is a separate background natural mortality term for age-1 mortality (the φ term in Eqn. 1.11). Annual age-1 natural mortality is estimated on a log scale independently for all years, except the last two years which are set equal to natural mortality in the third-to-last year. In the likelihood function, age-1 natural mortality is fit using a prior estimate provided in the ADMB data file (from Rybicki and Keller 1978). As with Lake Superior natural mortality, the prior is specified using Pauly's equation (Eqn. 1.3).

MH-1 and MH-2 natural mortality

Natural mortality in Lake Huron is unique in that it varies according to length. Age-and year-specific M is modeled as

$$M_{y,a}^{B} = e^{b_0} \left(\frac{1}{10} L_{y,a}\right)^{b_1} \left(\frac{1}{10} A_y\right)^{b_2} K_{y,a}^{b_3} \tag{2.6.2}$$

where b_0, b_1 and b_2 are coefficients for age- and year-specific length at age ($L_{y,a}$), year-specific von Bertalanffy asymptotic size (A_y) and age- and year-specific von Bertalanffy Brody growth coefficient K ($K_{y,a}$). $b_0 - b_3$ are estimated parameters (with specified priors in the likelihood function), while year-specific values for A and year- and age-specific values for K are provided in the data file.

As in the other areas, lamprey mortality by year and age is estimated externally to the model. Total mortality is the sum of background mortality and lamprey mortality (Eqn. 2.6.1). The natural mortality that is compared in the objective function to the prior estimate (based on Pauly's equation) is the average M^B for ages 5-15.

Annual age-1 natural mortality is estimated on a log scale independently for all years except the last five years in MH-1 and three years in MH-2; these non-estimated mortalities are assumed equal to the mean of the last three estimated years in each case. In the likelihood function, age-1 natural mortality is assumed to follow a normal distribution centered on zero.

2.7 Refuges

MM-123 and MM-67 (2012) Refuge

Population dynamics within the refuge are run parallel to those outside, but the individuals within the refuge suffer only natural mortality (the sum of the estimated background mortality and lamprey mortality). Refuge recruitment in year y, R_y^*, is

$$R_y^* = \nu R_y \tag{2.7.1}$$

where R_y is total recruitment, found using the stocking data (as in Eqn. 2.3.3), and ν is the proportion of stocked individuals that end up in the refuge. ν is not estimated, but provided in the data file as part of the migration matrix. Refuge population size is set in the first model year using the initial recruitment in year 1. The abundance in subsequent age classes in the initial year is found assuming exponential survival

$$N_{y+1,a+1} = N_{y,a} e^{-M} \tag{2.7.2}$$

2.8 Standard deviation priors

In some areas the standard deviation priors that are used in the objective function are provided directly in the data file, but in other cases a variance ratio is used along with a common standard deviation estimated within the model.

MI-5 MI-6 and MI-7 (2012) Standard deviation priors

Lake Superior areas use non-estimated standard deviations in the objective functions that are specified in the ADMB data file.

MM-123, MM-4 and MM-5 Standard deviation priors

The relative size ρ for each standard deviation is provided in the data file and a common standard deviation is estimated within the model

$$\sigma_q = \sigma \rho_q \tag{2.8.1}$$

where σ_q is the standard deviation for parameter q used in the objective function, σ is the model-estimated common standard deviation and ρ_q is the externally provided relative scalar for the standard deviation of parameter q.

MH-1 and MH-2 Standard deviation priors

The Lake Huron areas are set up to use variance ratios, though the common σ is fixed, so each σ_p is also externally fixed.

2.9 Likelihood and prior functions employed in the models

Normal log likelihood

For a single observation or value the log likelihood or log prior is

$$n(\mu, \sigma | x) = -\ln \sigma - \frac{(x - \mu)^2}{2\sigma^2} \tag{2.9.1}$$

where μ is the mean, σ is the standard deviation and x are the data. For multiple independent values the log likelihood or log prior is the sum of Eqn. 2.9.1 over the data.

Lognormal log likelihood or log prior

A single value has the log-likelihood or log prior

$$l(\mu, \sigma | x) = -\ln \sigma - \frac{(\ln x - \mu)^2}{2\sigma^2} - \ln x \tag{2.9.2}$$

where μ is the log mean, σ is the standard deviation and x are the data. The log-likelihood or log prior for multiple independent values is the sum of Eqn. 2.9.2 over the observations. Often $\ln x$ is a constant that does not affect at what parameter values the likelihood or prior is maximized and so can be ignored. However, in some cases x is not data but rather an estimated parameter or process error; in this case the $\ln x$ term should be included in the likelihood.

Multinomial log likelihood

The multinomial log likelihood is

$$m(\hat{p} | p, N) = N \sum_{k=1}^{k} (p_k \log \hat{p}_k) \tag{2.9.3}$$

where N is the sample size, p_k the k^{th} observed proportion and $\hat{p}_k$ the k^{th} estimated proportion.

Note on likelihood functions

The difference between the normal and lognormal log likelihoods/priors bears special mention. As noted above, the lognormal uses the difference between the log of the data and the log-scale mean and also includes a separate term for the log of the data. Many likelihood functions used in these models assume a normal distribution and use the difference between the log of the data and the log-scale mean, but do not include a separate term for the log of the data. For true likelihoods for fixed data the distinction is not important because the data are invariant so the term for the log of the data is a constant. However, when the x in Eqns. 2.9.1 or 2.9.2 is estimated the distinction is important. The likelihoods are minimized at different values and have different influences on the overall point estimates. From a Bayesian perspective different priors are specified regarding what the relative likelihood of different parameters are. A technical issue is that a mismatch between what scale a parameter is specified on and the scale for which the prior is specified could cause issues for MCMC procedures that develop posterior distributions. For example if for a parameter P a lognormal prior component is included but the actual parameter estimated is $\log(P)$ this would be a mismatch.

2.10 Likelihood Tables

Below are tables describing the likelihood components for the different stock assessment models. D/I distinguishes data-based components (D) from informative priors (I), λ is the weighting factor (if present), L is the form of the likelihood component, and μ, σ and x describe those parameters in the normal or lognormal likelihoods. Similar descriptions are not provided for multinomial proportions-at-age because $\hat{p}$ is always the predicted proportion-at-age for the fishery or survey, p is always the observed proportion-at-age and N is always the sample size. All terms in the normal or lognormal log-likelihoods are on a log scale; for example, if the μ column reads "estimated catch" this means estimated catch on a log scale. Some of the likelihood components are coded as normal distributions or as a sum of squares; however if these are equivalent to a different distribution (e.g., lognormal) this is noted.

MI-5 objective function

Term	D/I	λ	L	μ	σ	x
Commercial catch	D	1	$^{1}n(\mu,\sigma\|x)$	Estimated catch	External fixed	Catch data
Recreational catch	D	1	$^{1}n(\mu,\sigma\|x)$	Estimated catch	External fixed	Catch data
Large mesh CPUE	D	1	$^{1}n(\mu,\sigma\|x)$	Estimated CPUE	External fixed	CPUE data
Graded mesh CPUE	D	1	$^{1}n(\mu,\sigma\|x)$	Estimated CPUE	External fixed	CPUE data
Commercial proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-
Recreational proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-
Large mesh survey	D	1	$m(\hat{p}\|p,N)$	-	-	-

Term	D/I	λ	L	μ	σ	x
proportions-at-age						
Graded mesh survey proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-
Natural mortality	I	1	$n(\mu,\sigma\|x)$	Pauly Eqn.	External fixed	Estimate
Recreational q dev vector	I	0.01	[1]$n(\mu,\sigma\|x)$	0	External fixed	Dev vector
Commercial q dev vector	I	0.01	[1]$n(\mu,\sigma\|x)$	0	External fixed	Dev vector
Recruitment dev vector	I	0.01	[1]$\sum x^2$	-	-	Dev vector
Commercial selectivity k RW	I	-	$n(\mu,\sigma\|x)$	0	External fixed	RW vector
Large Mesh selectivity k RW	I	-	$n(\mu,\sigma\|x)$	0	External fixed	RW vector

[1]Equivalent to lognormal distribution up to a constant.

MI-6 objective function

Term	D/I	λ	L	μ	σ	x
Commercial catch	D	1	[1]$n(\mu,\sigma\|x)$	Estimated catch	External fixed	Catch data
Recreational catch	D	1	[1]$n(\mu,\sigma\|x)$	Estimated catch	External fixed	Catch data
Large mesh CPUE	D	1	[1]$n(\mu,\sigma\|x)$	Estimated CPUE	External fixed	CPUE data
Graded mesh CPUE	D	1	[1]$n(\mu,\sigma\|x)$	Estimated CPUE	External fixed	CPUE data
Commercial proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-
Recreational proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-
Large mesh	D	1	$m(\hat{p}\|p,N)$	-	-	-

Term	D/I	λ	L	μ	σ	x
survey proportions-at-age						
Graded mesh survey proportions-at-age	D	1	$m(\hat{p}\|p, N)$	-	-	-
Natural mortality	I	1	$n(\mu, \sigma\|x)$	Pauly Eqn.	External fixed	Estimate
Recreational q dev vector	I	0.01	[1]$n(\mu, \sigma\|x)$	0	External fixed	Dev vector
Commercial q dev vector	I	0.01	[1]$n(\mu, \sigma\|x)$	0	External fixed	Dev vector
Large mesh survey q	I	-	$n(\mu, \sigma\|x)$	Prior	External fixed	Estimate
Recruitment RW vector	I	-	$\frac{\sum x^2}{2\sigma^2}$	-	-	RW vector
Commercial selectivity k RW	I	-	$n(\mu, \sigma\|x)$	0	External fixed	RW vector
Large Mesh selectivity k RW	I	-	$n(\mu, \sigma\|x)$	0	External fixed	RW vector

[1]Equivalent to lognormal distribution up to a constant.

MI-7 (2012) objective function

Term	D/I	λ	L	μ	σ	x
Commercial catch	D	1	[1]$n(\mu, \sigma\|x)$	Estimated catch	External fixed	Catch data
Recreational catch	D	1	[1]$n(\mu, \sigma\|x)$	Estimated catch	External fixed	Catch data
Large mesh CPUE	D	1	[1]$n(\mu, \sigma\|x)$	Estimated CPUE	External fixed	CPUE data
Graded mesh CPUE	D	1	[1]$n(\mu, \sigma\|x)$	Estimated CPUE	External fixed	CPUE data

Term	D/I	λ	L	μ	σ	x
Commercial proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-
Recreational proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-
Large mesh survey proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-
Graded mesh survey proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-
Natural mortality	I	1	$n(\mu,\sigma\|x)$	Pauly Eqn.	External fixed	Estimate
Recreational q dev vector	I	0.01	[1] $n(\mu,\sigma\|x)$	0	External fixed	Dev vector
Commercial q dev vector	I	0.01	[1] $n(\mu,\sigma\|x)$	0	External fixed	Dev vector
Recruitment dev vector	I	-	$\frac{\sum x^2}{2\sigma^2}$	0	External fixed	Dev vector
Commercial selectivity k	I	-	$n(\mu,\sigma\|x)$	0	External fixed	RW vector
Recreational selectivity k	I	-	$n(\mu,\sigma\|x)$	0	External fixed	RW vector
Large Mesh selectivity k	I	-	$n(\mu,\sigma\|x)$	0	External fixed	RW vector

[1]Equivalent to lognormal distribution up to a constant.

MM-123 objective function

Term	D/I	λ	L	μ	σ	x
Commercial catch	D	1	[1] $n(\mu,\sigma\|x)$	Estimated catch	Variance ratio	Catch data
Recreational	D	1	[1] $n(\mu,\sigma\|x)$	Estimated catch	Variance	Catch data

Term	D/I	λ	L	μ	σ	x
catch					ratio	
Survey CPUE	D	1	$^{1}n(\mu,\sigma\|x)$	Estimated CPUE	External fixed	CPUE data
Commercial proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-
Recreational proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-
Survey proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-
Natural mortality	I	1	$n(\mu,\sigma\|x)$	Pauly Eqn.	External fixed	Estimate
Age-1 Natural mortality	I	-	$n(\mu,\sigma\|x)$	Prior	Variance ratio	Estimates
Recreational q RW	I	1	$n(\mu,\sigma\|x)$	0	Variance ratio	RW vector
Commercial q RW	I	1	$n(\mu,\sigma\|x)$	0	Variance ratio	RW vector

[1]Equivalent to lognormal distribution up to a constant.

MM-4 objective function

Term	D/I	λ	L	μ	σ	x
Commercial catch	D	1	$^{1}n(\mu,\sigma\|x)$	Estimated catch	Variance ratio	Catch data
Recreational catch	D	1	$^{1}n(\mu,\sigma\|x)$	Estimated catch	Variance ratio	Catch data
Survey CPUE	D	1	$^{1}n(\mu,\sigma\|x)$	Estimated CPUE	External fixed	CPUE data
Commercial proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-

Term	D/I	λ	L	μ	σ	x
Recreational proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-
Survey proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-
Natural mortality	I	1	$n(\mu,\sigma\|x)$	Pauly Eqn.	External fixed	Estimate
Age-1 Natural mortality	I	-	$n(\mu,\sigma\|x)$	Prior	Variance ratio	Estimates
Recreational q RW	I	1	$n(\mu,\sigma\|x)$	0	Variance ratio	RW vector
Commercial q RW	I	1	$n(\mu,\sigma\|x)$	0	Variance ratio	RW vector
Survey selectivity k RW	I	1	$n(\mu,\sigma\|x)$	0	Variance ratio	RW vector

[1]Equivalent to lognormal distribution up to a constant.

MM-5 objective function

Term	D/I	λ	L	μ	σ	x
Commercial catch	D	1	[1]$n(\mu,\sigma\|x)$	Estimated catch	Variance ratio	Catch data
Recreational catch	D	1	[1]$n(\mu,\sigma\|x)$	Estimated catch	Variance ratio	Catch data
Survey CPUE	D	1	[1]$n(\mu,\sigma\|x)$	Estimated CPUE	External fixed	CPUE data
Commercial proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-
Recreational proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-
Survey	D	1	$m(\hat{p}\|p,N)$	-	-	-

Term	D/I	λ	L	μ	σ	x
proportions-at-age						
Natural mortality	I	1	$n(\mu, \sigma \mid x)$	Pauly Eqn.	External fixed	Estimate
Age-1 Natural mortality	I	-	$n(\mu, \sigma \mid x)$	Prior	Variance ratio	Estimates
Recreational q RW	I	1	$n(\mu, \sigma \mid x)$	0	Variance ratio	RW vector
Commercial q RW	I	1	$n(\mu, \sigma \mid x)$	0	Variance ratio	RW vector
Survey selectivity k RW 1	I	1	$n(\mu, \sigma \mid x)$	0	Variance ratio	RW vector
Survey selectivity k RW 2	I	1	$n(\mu, \sigma \mid x)$	0	Variance ratio	RW vector

[1]Equivalent to lognormal distribution up to a constant.

MM-67 (2012) objective function

Term	D/I	λ	L	μ	σ	x
Commercial catch	D	1	[1]$n(\mu, \sigma \mid x)$	Estimated catch	External fixed	Catch data
Recreational catch	D	1	[1]$n(\mu, \sigma \mid x)$	Estimated catch	External fixed	Catch data
Survey CPUE	D	1	[1]$n(\mu, \sigma \mid x)$	Estimated CPUE	External fixed	CPUE data
Recreational proportions-at-age	D	1	$m(\hat{p} \mid p, N)$	-	-	-
Survey proportions-at-age	D	1	$m(\hat{p} \mid p, N)$	-	-	-
Natural mortality	I	1	$n(\mu, \sigma \mid x)$	Pauly Eqn.	External fixed	Estimate

Term	D/I	λ	L	μ	σ	x
Age-1 Natural mortality	I	-	$n(\mu, \sigma \| x)$	Prior	External fixed	Estimates
Recreational q dev vector	I	0.01	[1]$n(\mu, \sigma \| x)$	0	External fixed	Dev vector

[1]Equivalent to lognormal distribution up to a constant.
Note: commercial proportions at age are not included in the likelihood.

MH-1 objective function

Term	D/I	λ	L	μ	σ	x
Commercial catch	D	-	[1]$n(\mu, \sigma \| x)$	Estimated catch	Variance ratio with fixed σ	Catch data
Recreational catch	D	-	[1]$n(\mu, \sigma \| x)$	Estimated catch	Variance ratio with fixed σ	Catch data
Survey CPUE	D	-	[1]$n(\mu, \sigma \| x)$	Estimated CPUE	External fixed	CPUE data
Commercial proportions-at-age	D	-	$m(\hat{p} \| p, N)$	-	-	-
Recreational proportions-at-age	D	-	$m(\hat{p} \| p, N)$	-	-	-
Survey proportions-at-age	D	-	$m(\hat{p} \| p, N)$	-	-	-
Fall mature female proportions-at-age	D	-	$m(\hat{p} \| p, N)$	-	-	-
Commercial wild ratio-at-age	D	-	$m(\hat{p} \| p, N)$	-	-	-
Survey wild ratio-at-age	D	-	$m(\hat{p} \| p, N)$	-	-	-

Term	D/I	λ	L	μ	σ	x
Recreational wild ratio-at-age	D	-	$m(\hat{p}\|p, N)$	-	-	-
Commercial q	I	-	$l(\mu, \sigma\|x)$	Prior (variable by year)	Variance ratio with fixed σ	Estimates
Survey q	I	-	$l(\mu, \sigma\|x)$	Prior (variable by year)	Variance ratio with fixed σ	Estimates
Recreational q RW	I	-	$l(\mu, \sigma\|x)$	0	Variance ratio with fixed σ	RW vector
Commercial selectivity σ	I	-	$l(\mu, \sigma\|x)$	Prior (variable by year)	Variance ratio with fixed σ	Estimates
Recreational selectivity σ	I	-	$l(\mu, \sigma\|x)$	Prior (variable by year)	[2]Variance ratio with fixed σ	Estimates
Survey selectivity σ	I	-	$l(\mu, \sigma\|x)$	Prior (variable by year)	[2]Variance ratio with fixed σ	Estimates
Commercial selectivity μ	I	-	$l(\mu, \sigma\|x)$	Prior	[3]Fixed in code	Estimate
Recreational selectivity μ	I	-	$l(\mu, \sigma\|x)$	Prior	[3]Fixed in code	Estimate
Survey selectivity μ	I	-	$l(\mu, \sigma\|x)$	Prior	[3]Fixed in code	Estimate
Natural mortality parameter b_0	I	-	$l(\mu, \sigma\|x)$	Prior	[4]External fixed	Estimate
Natural mortality parameter b_1	I	-	$l(\mu, \sigma\|x)$	Prior	[4]External fixed	Estimate
Natural mortality	I	-	$l(\mu, \sigma\|x)$	Prior	[4]External fixed	Estimate

Term	D/I	λ	L	μ	σ	x
parameter b_2						
Natural mortality parameter b_3	I	-	$l(\mu, \sigma \| x)$	Prior	[4]External fixed	Estimate
Average natural mortality	I	-	$l(\mu, \sigma \| x)$	Prior	[4]External fixed	Estimate
Age-1 natural mortality	I	-	$l(\mu, \sigma \| x)$	0	Variance ratio with fixed σ	Estimates

[1]Equivalent to lognormal distribution up to a constant.
[2]In some years σ is divided by 2 within the likelihood function.
[3]The recreational and survey selectivities share a common fixed σ.
[4]The length-based natural mortality equation parameters and the average natural mortality estimate share a common fixed σ.

MH-2 objective function

Term	D/I	λ	L	μ	σ	x
Commercial catch	D	-	[1]$n(\mu, \sigma \| x)$	Estimated catch	Variance ratio with fixed σ	Catch data
Recreational catch	D	-	[1]$n(\mu, \sigma \| x)$	Estimated catch	Variance ratio with fixed σ	Catch data
Survey CPUE	D	-	[1]$n(\mu, \sigma \| x)$	Estimated CPUE	External fixed	CPUE data
Commercial proportions-at-age	D	-	$m(\hat{p} \| p, N)$	-	-	-
Recreational proportions-at-age	D	-	$m(\hat{p} \| p, N)$	-	-	-
Survey proportions-at-age	D	-	$m(\hat{p} \| p, N)$	-	-	-

Term	D/I	λ	L	μ	σ	x
Commercial wild ratio-at-age	D	-	$m(\hat{p}\|p,N)$	-	-	-
Survey wild ratio-at-age	D	-	$m(\hat{p}\|p,N)$	-	-	-
Recreational wild ratio-at-age	D	-	$m(\hat{p}\|p,N)$	-	-	-
Commercial q	I	-	$l(\mu,\sigma\|x)$	Prior (variable by year)	Variance ratio with fixed σ	Estimates
Survey q	I	-	$l(\mu,\sigma\|x)$	Prior (variable by year)	[2]Variance ratio with fixed σ	Estimates
Recreational q RW	I	-	$l(\mu,\sigma\|x)$	0	Variance ratio with fixed σ	RW vector
Commercial selectivity σ	I	-	$l(\mu,\sigma\|x)$	Prior (variable by year)	Variance ratio with fixed σ	Estimates
Recreational selectivity σ	I	-	$l(\mu,\sigma\|x)$	Prior (variable by year)	[2]Variance ratio with fixed σ	Estimates
Survey selectivity σ	I	-	$l(\mu,\sigma\|x)$	Prior (variable by year)	Variance ratio with fixed σ	Estimates
Commercial selectivity μ	I	-	$l(\mu,\sigma\|x)$	[3]Prior	[3]Fixed in code	Estimate
Recreational selectivity μ	I	-	$l(\mu,\sigma\|x)$	[3]Prior	[3]Fixed in code	Estimate
Survey selectivity μ	I	-	$l(\mu,\sigma\|x)$	[3]Prior	[3]Fixed in code	Estimate
Natural mortality parameter b_0	I	-	$l(\mu,\sigma\|x)$	Prior	[4]External fixed	Estimate

Term	D/I	λ	L	μ	σ	x
Natural mortality parameter b_1	I	-	$l(\mu,\sigma\|x)$	Prior	[4]External fixed	Estimate
Natural mortality parameter b_2	I	-	$l(\mu,\sigma\|x)$	Prior	[4]External fixed	Estimate
Natural mortality parameter b_3	I	-	$l(\mu,\sigma\|x)$	Prior	[4]External fixed	Estimate
Average natural mortality	I	-	$l(\mu,\sigma\|x)$	Prior	[3]External fixed	Estimate
Age-1 natural mortality	I	-	$l(\mu,\sigma\|x)$	0	Variance ratio with fixed σ	Estimates

[1]Equivalent to lognormal distribution up to a constant.
[2]In some years σ is divided by 2 within the likelihood function.
[3]The commercial, recreational and survey selectivities share a common fixed μ and σ.
[4]The length-based natural mortality equation parameters and the average natural mortality estimate share a common fixed σ.

3 Lake whitefish assessment models

A description of submodels for each lake whitefish assessment area as they stood at the end of the 2014 assessment cycle, including any important differences between these models and the approach described by Caroffino and Lenart (2011), is given below.

3.1 Selectivity

WFS-04 selectivity

Selectivity in WFS-04 is implemented using logistic functions which are length- rather than age-based (as in lake trout areas MH-1 and MH-2). A time-varying double logistic function is used for the gillnet fishery. Non-standardized selectivity by age and year ($S^*_{a,y}$) is

$$S^*_{a,y} = \frac{1}{1+e^{-\theta_1(L_a-\theta_{2,y})}}\left(1-\frac{1}{1+e^{-\theta_3(L_a-\theta_4)}}\right) \tag{3.1.1}$$

where L_a is length at age a and θ_1, θ_2, θ_3 and θ_4 are the logistic parameters. The $\theta_{2,y}$ parameter is time-varying according to a random walk such that

$$\theta_{2,y} = \begin{cases} e^{k_{y_1}} & \text{if } y = y_1 \\ e^{\log(\theta_{2,y-1})+k_y} & \text{if } y_1 < y \leq y_{MAX} \end{cases} \tag{3.1.2}$$

where k is a direct estimate for the log of θ_2 in the first year y_1 and a log-scale deviation estimate for each year (y) after the first year. The same approach, including the θ_2 random walk, is taken for the trap net fishery, only a single logistic function is used

$$S^*_{a,y} = \frac{1}{1+e^{-\theta_1(L_a-\theta_{2,y})}} \tag{3.1.3}$$

so maximum selectivity is achieved in the largest age class. The $\theta_{2,y}$ random walk sequence for both the gillnet and trap net fisheries begin in 1986 and end in 2012. Standardization is in both cases carried out by choosing a reference length external to the model so the final selectivity at age a in year y, $S_{a,y}$, is

$$S_{a,y} = \frac{d(L_a;\theta_1,\theta_2,\theta_3,\theta_4,y)}{d(r^L;\theta_1,\theta_2,\theta_3,\theta_4,y)} \tag{3.1.4}$$

where $d()$ is the double logistic function, L_a is length at age a, and r^L is the reference length. The denominator is re-calculated each year to provide the standardization. The single logistic trap net fishery is standardized in the same way as Eqn. 3.1.4, just without the θ_3 and θ_4 parmaters. The reference lengths are 500 mm for the Gillnet selectivity and 530 for the trap net selectivity. The k random walk parameter vectors for both trap nets and gillnets are included in the likelihood function and assumed to follow a normal distribution.

Selectivity information

Description	Model Years
GN random walk	1987-2012

TN random walk	1987-2012

WFS-05 selectivity

Selectivity in WFS-05 uses an age-based gamma function (Eqn. 2.1.2) to estimate selectivity with a rate parameter that varies through time according to a random walk (Eqn. 2.1.3). Standardization is via the max function (Eqn. 2.1.4). This is the same approach as is implemented in the lake trout MI-5 area, save the further adjustments that are made to selectivity in that area (see the MI-5 selectivity section for a complete description). The k parameter random walk vectors for both trap nets and gillnets are included in the likelihood function and assumed to follow a normal distribution.

Selectivity information

Description	Model Years
GN random walk	1987-2012
TN random walk	1987-2012

WFS-07 selectivity

Selectivity in WFS-07 uses length-based logistic functions for the gillnet and trap net fisheries that vary through time according to a random walk. The method is almost identical to that used in WFS-04, except that the trap net fishery also uses a double logistic function as opposed to a single logistic (i.e., replacement of Eqn. 3.1.3 with Eqn. 3.1.1 in the WFS-04 area makes these two approaches identical). The time varying random walk component is $\theta_{2,y}$ (Eqn. 3.1.1) and standardization uses an externally determined reference length of 518 mm for both gillnets and trap nets. The k parameter vectors for both trap nets and gillnets are included in the likelihood function and assumed to follow a normal distribution.

Selectivity information

Description	Model Years
GN random walk	1977-2012
TN random walk	1977-2012

WFS-08 selectivity

The WFS-08 assessment employs double logistic functions to estimate selectivity for both the gillnet and the trap net fisheries, though unlike in areas WFS-04 and WFS-07 the function is age-based rather than length-based (Eqn. 1.5). The time varying random walk component is $\beta_{2,y}$ (Eqn. 1.5). Trap net selectivity from 1981 (the first model year) through 1997 is time-invariant so

$$\beta_{2,y} = \begin{cases} e^{k_{y_1}} & \text{if } 1981 \leq y \leq 1996 \\ e^{\log(\beta_{1,y-1})+k_y} & \text{if } y > 1996 \end{cases} \tag{3.1.5}$$

where k is a log-scale estimate for β_1 in years prior to 1997 and a log-scale deviation estimate for each year (y) after that. Selectivity is standardized using reference ages

$$S_{a,y} = \frac{d(a; \beta_1, \beta_2, \beta_3, \beta_4, y)}{d(r^A; \beta_1, \beta_2, \beta_3, \beta_4, y)} \tag{3.1.6}$$

where a is age. The reference age r^A for the gillnet fishery is 7 and for the trap net fishery is 6. The k random walk parameter vectors for both trap nets and gillnets (for the years that vary) are included in the likelihood function and assumed to follow a normal distribution.

Selectivity information

Description	Model Years
GN random walk	1982-2012
TN random walk	1997-2012

WFM-01 selectivity

Selectivity in the WFM-01 assessment, for which there is a trap net fishery only, uses a length-based double logistic model with a time-varying parameter ($\theta_{2,y}$; Eqn. 3.1.1) that varies according to a random walk (Eqn. 3.1.2). Standardization uses an externally determined reference length (Eqn. 3.1.4) of 483 mm. The k parameter vector for the random walk is included in the likelihood function and assumed to follow a normal distribution.

Selectivity information

Description	Model Years
GN random walk	1977-2012
TN random walk	1977-2012

WFM-02 selectivity

Selectivity in WFM-02 employs length-based logistic functions. A double logistic function (3.1.1) is used for gillnet selectivity and a single logistic (3.1.3) for trap net selectivity. The parameter $\theta_{2,y}$ (Eqns. 3.1.1 and 3.1.3) varies over time according to a quadratic function

$$\theta_{2,y} = \omega_0 + \delta_y + \omega_1(y - y_1) + \omega_2(y - y_1)^2 \tag{3.1.7}$$

where ω_0, ω_1 and ω_2 are quadratic function parameters. Standardization uses the max function, but is based on the maximum selectivity in the first year (as opposed to other cases where selectivity in each year is standardized according to the maximum value in each year e.g., Eqn. 3.1.4). The equation is

$$S_{a,y} = \frac{d(L_a; \theta_1, \theta_2, \theta_3, \theta_4, y)}{d(r^L; \theta_1, \theta_2, \theta_3, \theta_4, y_1)} \tag{3.1.8}$$

where d is the double logistic function, r^L is the reference length, L_a is length at age, y is year, θ_1, θ_2, θ_3 and θ_4 are the double logistic parameters, and y_1 indicates the first model year. The reference length for gillnets is 485 mm and for trap nets is 410.

Some modifications are made to selectivity in the trap net fishery. The 1988 and 1989 selectivity adjustments (Eqn. 3.1.7) are set equal to the 1987 selectivity and the adjustment during 2004-2010 is set equal to the 2003 adjustment. This does not make selectivities identical during those years, however, because length-at-age (L_a in Eqn. 3.1.1) changes each year.

Selectivity information

Description	Model Years
GN quadratic	1986-1989, 1993-2012
TN quadratic	1986-1987, 1991-2003, 2012

WFM-03 selectivity

Selectivity in WFM-03 is estimated in the same manner as in WFM-02, except there are no modifications made to the annual adjustment term (Eqn. 3.1.7) for particular years. The reference length for gillnet selectivity is 485 and for trap net selectivity is 455.

Selectivity information

Description	Model Years
GN quadratic	1986-2012
TN quadratic	1986-2012

WFM-04 selectivity

Selectivity in the WFM-04 assessment uses a length-based double logistic function for the gillnet fishery and a length-based single logistic selectivity for the trap net fishery (Eqns. 3.1.1 and 3.1.3). The $\theta_{2,y}$ parameters (Eqns. 3.1.1, 3.1.3) in the logistic functions vary according to a random walk as in Eqn. 3.1.2. Standardization is based on selectivity at a reference length in the first model year (Eqn. 3.1.8). The reference lengths are 527 mm for gillnet selectivity and 493 for trap net selectivity. The k parameter vector for the random walk is included in the likelihood function and assumed to follow a normal distribution.

Selectivity information

Description	Model Years
GN random walk	1982-2012

TN random walk	1990-2012

WFM-05 selectivity

Selectivity in the WFM-05 model proceeds as in WFM-04. The reference length r^L for both the gillnet and trap net selectivity is 508 mm. The k parameter vector for the random walk is included in the likelihood function and assumed to follow a normal distribution.

Selectivity information

Description	Model Years
GN random walk	1982-2012
TN random walk	1982-2012

WFM-06 selectivity

Selectivity in WFM-06 uses a length-based double logistic selectivity for the gillnet fishery and a length-based single logistic selectivity for the trap net fishery (Eqns. 3.1.1 and 3.1.3). The $\theta_{2,y}$ parameter (Eqns. 3.1.1, 3.1.3) in the logistic functions varies according to a random walk as in Eqn. 3.1.2, however θ_2 for the gillnet fishery is invariant before 1994, the first year of the random walk. Standardization is based on selectivity at a reference length in the first model year (Eqn. 3.1.8). The reference lengths are 527 mm for gillnet selectivity and 493 for trap net selectivity. The k parameter vector for the random walks is included in the likelihood function and assumed to follow a normal distribution.

Selectivity information

Description	Model Years
GN random walk	1994-2012
TN random walk	1986-2012

WFM-08 selectivity

Selectivity in WFM-08 uses a length-based logistic selectivity for the trap net fishery (Eqn. 3.1.3; this is the only fishery). The $\theta_{2,y}$ parameter (Eqn. 3.1.3) in the logistic function varies according to a random walk (Eqn. 3.1.2). Standardization is based on selectivity at a reference length of 495 mm in the first model year (Eqn. 3.1.8). The k parameter vector for the random walk is included in the likelihood function and assumed to follow a normal distribution.

Selectivity information

Description	Model Years
TN random walk	1986-2012

WFH-1-4 selectivity

Selectivity in the WFH-1-4 assessment uses length-based double logistic functions for both the gillnet and trap net fisheries (Eqn. 3.1.1). The $\theta_{2,y}$ parameter in the logistic function varies according to a random walk as in Eqn. 3.1.2, however $\theta_{2,y}$ for the gillnet fishery is directly estimated prior to 1983, the first year of the random walk. Standardization is similar to that used in Eqn. 3.1.8 except the denominator varies by year as opposed to being locked in the first year (Eqn. 3.1.4). The reference lengths r^L used for selectivity are 483 mm for the gillnet fishery and 450 for the trap net fishery. The k parameter vector for the random walks are included in the likelihood function and assumed to follow a normal distribution.

Selectivity information

Description	Model Years
GN random walk	1983-2012
TN random walk	1977-2012

WFH-05 selectivity

Selectivity-at-age for gillnets and trap nets in WFH-05 is estimated using a length-based lognormal distribution. Non-standardized selectivity S^* at age a in year y is

$$S^*_{a,y} = \frac{1}{\sigma_y L_a \sqrt{2\pi}} \exp\left(-\frac{(\ln(L_a) - \mu_b)^2}{2\sigma_y^2}\right) \qquad (3.1.9)$$

where σ_y is the lognormal standard deviation in year y, L_a is the average length at age a and μ_b is the lognormal mean. μ varies by time block b in the trap net fishery, but is constant in the gillnet fishery. The time blocks for μ in the trap net fishery are 1981-1986, 1987-2002 and 2003-2012. The σ parameter varies by year for both the gillnet fishery and the trap net fishery, though the methods are different. σ for the gillnet fishery varies according to a random walk (Eqn. 3.1.2) after 1983 (prior to this a single value is estimated, making selectivity invariant). σ for the trap net fishery in all years is estimated using the following equation

$$\sigma_y = e^{\phi e^{\lambda_y}} \qquad (3.1.10)$$

Where ϕ is an estimated log-scale baseline value and λ_y are the log-scale annual deviations to the baseline value.

The selectivity is standardized using the lognormal pdf at the mean during each time block μ_b

$$S_{REF,y,b} = \frac{1}{\sigma_y e^{\mu_b} \sqrt{2\pi}} \qquad (3.1.11)$$

and

$$S_{a,y,b} = \frac{S^*_{a,y,b}}{S_{REF,y,b}} \qquad (3.1.12)$$

The trap net random walk k is not included in the likelihood function, but the vector λ is assumed to follow a normal distribution with a mean of 0 and ϕ is assumed normal with a prior of 2.35.

Selectivity information

Description	Model Years
GN random walk	1984-2012
TN annual variability	1981-2012

3.2 Catchability

WFS-04 and WFS-05 catchability

Catchability for the gillnet and trap net fisheries in WFS-04 varies throughout the time series according to a random walk (Eqn. 2.2.5). The random walk vector η (excluding η_{y_1}) is included in the likelihood function for both the gillnet fishery and the trap net fishery and is assumed to follow a normal distribution.

Catchability information

Description	Model Years
GN random walk	1987-2012
TN random walk	1987-2012

WFS-07 catchability

Catchability for the gillnet and trap net fisheries in WFS-07 varies throughout the time series according to a random walk (Eqn. 2.2.5). The random walk vector η (excluding η_{y_1}) is included in the likelihood function for both the gillnet fishery and the trap net fishery and is assumed to follow a normal distribution.

Catchability information

Description	Model Years
GN random walk	1977-2012
TN random walk	1977-2012

WFS-08 catchability

Catchability for gillnet and trap net fisheries in WFS-08 are estimated using vectors that are constrained to sum to zero and are added to the mean q to produce the final annual catchabilities

$$q_y = e^{\bar{q}+\nu_y} \tag{3.2.1}$$

where $\bar{q}$ is the estimated mean value for the log of q and ν_y are the annual deviations. There was no trap net fishery from 1987 to 1995 so catchability was not estimated for those years. The dev vectors ν for both fisheries are included in the likelihood function and assumed to be normally distributed (though equivalent to a lognormal assumption; see section 2.9).

Catchability information

Description	Model Years
GN dev vector	1981-2012
TN dev vector	1981-2012

WFM-01 catchability

Catchability for the WFM-01 trap net fishery varies throughout the time series according to a random walk (Eqn. 3.2.2). The random walk vector η (excluding η_{y_1}) is included in the likelihood function for the trap net fishery and is assumed to follow a normal distribution.

Catchability information

Description	Model Years
TN random walk	1976-2012

WFM-02 catchability

Catchability for gillnets and trap nets in WFM-02 varies throughout the time series according to a random walk (Eqn. 2.2.5). The random walk vector η (excluding η_{y_1}) is included in the likelihood function for both the gillnet fishery and the trap net fishery and is assumed to follow a normal distribution.

Catchability information

Description	Model Years
GN random walk	1987-2012
TN random walk	1987-2012

WFM-03 catchability

Catchability for gillnets and trap nets in WFM-03 vary over time according to a random walk (Eqn. 2.2.5). The random walk vector η (excluding η_{y_1}) is included in the likelihood function for both the gillnet fishery (separately) and the trap net fishery and is assumed to follow a normal distribution.

Catchability information

Description	Model Years
GN random walk	1987-2012
TN random walk	1987-2012

WFM-04 catchability

Catchability in WFM-04 follows a random walk for the trap and gillnet fisheries, but is adapted to include a gap where there was no trap net effort during 1986-1988. The beginning of the series uses the standard random walk (Eqn. 2.2.5)

$$q_y = \begin{cases} e^{\eta_{y_{1981}}} & \text{if } y = y_{1981} \\ e^{\log(q_{y-1})+\eta_y} & \text{if } y_{1981} < y \leq y_{1985} \end{cases} \tag{3.2.2}$$

but during the gap years 1990-1992 catchability is not estimated. For 1989, the first year after the gap, the random walk is continued but with a correction factor

$$q_y = \begin{cases} e^{\log(q_{1989})+2\eta_{1989}} & \text{if } y = y_{1989} \\ e^{\log(q_{y-1})+\eta_y} & \text{if } y_{1989} < y \leq y_{2012} \end{cases} \tag{3.2.3}$$

where η_{1989} is a freely estimated parameter. The random walk vector η (excluding $\eta_{y_{1981}}$) is included in the likelihood function for both the gillnet fishery and the trap net fishery (separately) and is assumed to follow a normal distribution. The factor of 2 that is multiplied by η_{1989} ensures that the scale of η_{1989} matches the rest of the η vector in the likelihood function.

Catchability information

Description	Model Years
GN random walk	1982-2012
TN random walk	1982-1986, 1989-2012

WFM-05 catchability

Catchabilities for gillnets and trap nets in WFM-05 vary over time according to a random walk (Eqn. 2.2.5). The random walk vector η (excluding η_{y_1}) is included in the likelihood function for both the gillnet fishery and the trap net fishery (separately) and is assumed to follow a normal distribution.

Catchability information

Description	Model Years
GN random walk	1982-2012
TN random walk	1982-2012

WFM-06 catchability

Catchability in WFM-06 follows a random walk for the trap and gillnet fisheries, but is adapted to include a gap where there was no gillnet effort during 1990-1992. The correction method is the same as is used for WFM-04 above (Eqns. 3.2.2 and 3.2.3). The random walk vector η (excluding η_{y_1}) is included in the likelihood function for both the gillnet fishery and the trap net fishery (separately) and is assumed to follow a normal distribution.

Catchability information

Description	Model Years
GN random walk	1986-1989, 1993-2012
TN random walk	1986-2012

WFM-08 catchability

Catchability for the trap net fishery in WFM-05 varies over time according to a random walk (Eqn. 2.2.5). The random walk vector η (excluding η_{y_1}) is included in the likelihood function and is assumed to follow a normal distribution.

Catchability information

Description	Model Years
TN random walk	1986-2012

WFH-1-4 catchability

Catchability for the WFH-1-4 gillnet and trap net fisheries vary through the time series according to a random walk (Eqn. 2.2.5). The random walk vectors η (excluding η_{y_1}) are included in the likelihood function (separately for each fishery) and assumed to follow a normal distribution.

Catchability information

Description	Model Years
GN random walk	1977-2012
TN random walk	1977-2012

WFH-05 catchability

Catchability for the WFH-05 gillnet and trap net fisheries vary through the time series according to a random walk (Eqn. 2.2.5). The random walk vectors η (excluding η_{y_1}) are included in the likelihood function (separately for each fishery) and assumed to follow a normal distribution.

Catchability information

Description	Model Years
GN random walk	1982-2012
TN random walk	1982-2012

3.3 Recruitment

WFS-04, WFS-05, WFS-07, WFS-08, WFM-01, WFM-02, WFM-03, WFM-04, WFM-05, WFM-06, WFM-08, WFH-1-4 recruitment

Recruitment in these assessment areas is estimated together with initial population abundance as a single vector of deviations constrained to sum to zero. Recruitment is

$$R_y = e^{\bar{P}+r_y} \tag{3.3.1}$$

where R is recruitment in year y, $\bar{P}$ is the average estimate for recruitment in all years together with initial population size and r_y is the annual deviation for recruitment. The r_y need not sum to zero because the zero constraint is on the entire vector, including the initial population size. r_y is not included in the likelihood function, but the difference between R_y and recruitment estimated using a Ricker model is included (see section 1 Eqns. 1.12-1.14 and objective function section below).

Recruitment occurs at age 4 in areas WFS-04, WFS-05, WFS-07, WFS-08, WFM-03 and WFH-1-4. It occurs at age 3 in areas WFM-01, WFM-02, WFM-04, WFM-05, WFM-06 and WFM-08.

WFH-05 recruitment

Recruitment in WFH-05 is similar to the other areas, where

$$R_y = e^{\bar{P}+r_y} \tag{3.3.2}$$

However, in the other whitefish management areas recruitment is estimated together with initial population abundance in a single dev vector. Here they are split. The r vector is not constrained to sum to zero (i.e., it is not a dev vector), and it is included in the likelihood function (and assumed to be normally distributed), along with the difference between R_y and recruitment estimated using a Ricker model. Recruitment occurs at age 3.

3.4 Numbers-at-age

WFS-04 numbers-at-age

The general approach to numbers-at-age in WFS-04 is consistent with the original 1998 model design, but there are some modifications for deriving the initial population estimates.

Age classes in the initial model year are estimated as model parameters in the same dev vector as recruitment

$$I_a = e^{\bar{P}+i_a} \tag{3.4.1}$$

where I_a is the initial estimated number of fish in age class a and i is the log-scale parameter deviation estimate. $\bar{P}$ is the average recruitment-initial population size. The i vector is not constrained to sum to zero because though it is part of a white noise vector of deviations it is combined with the recruitment vector r (Eqn. 3.3.1). i is not included in the likelihood function.

WFS-05 numbers-at-age

Initial population size in area WFS-05 is estimated in the same manner as in WFS-04.

WFS-07 numbers-at-age

Initial population size in area WFS-07 is estimated in the same manner as in WFS-04, however the plus group in the initial age composition is set to zero.

WFS-08 numbers-at-age

Initial population size in area WFS-08 is estimated in the same manner as in WFS-04.

WFM-01 numbers-at-age

Initial population size in area WFM-01 is estimated in the same manner as in WFS-04, however the last two age groups in the initial age composition are set equal to zero.

WFM-02 numbers-at-age

Initial population size in area WFM-02 is estimated in the same manner as in WFS-04, however the last four age groups in the initial age composition are set equal to zero.

WFM-03 numbers-at-age

Initial population size in area WFM-03 is estimated in the same manner as in WFS-04, however the last eight age groups in the initial age composition are set equal to zero.

WFM-04 numbers-at-age

Initial population size in area WFM-04 is estimated in the same manner as in WFS-04, however numbers-at-age in the last age class are set equal to numbers-at-age in the second to last age class.

WFM-05 numbers-at-age

Initial population size in area WFM-05 is estimated in the same manner as in WFS-04.

WFM-06 numbers-at-age

Initial population size in area WFM-06 is estimated in the same manner as in WFS-04.

WFM-08 numbers-at-age

Initial population size in area WFM-08 is estimated in the same manner as in WFS-04, however numbers-at-age in the last age class are set equal to numbers-at-age in the second to last age class.

WFH-1-4 numbers-at-age

Initial population size in area WFH-1-4 is estimated in the same manner as in WFS-04, however the last two age groups in the initial age composition are set equal to zero.

WFH-05 numbers-at-age

Initial age composition in WFH-05 is similar to the other areas, where

$$I_a = e^{\bar{P}+i_a} \qquad (3.4.2)$$

However, in the other whitefish management areas initial age composition is estimated together with recruitment in a single white noise vector of deviations. Here they are split. The i vector is not constrained to sum to zero because it is not a dev vector, and it is not included in the likelihood function. Such a parameterization is mathematically equivalent to estimating initial abundance at each of the ages as a free parameter, but may have numerical superiority because the scale is estimated separately from the relative magnitude for different ages. Initial abundances for the last five age groups are set to zero.

3.5 Catch-at-age

Unlike in the lake trout models, there is no variability in the catch-at-age calculations for lake whitefish; they are based on the original 1998 assessment model (see section 1).

3.6 Natural mortality

WFS-04, WFS-05, WFS-07, WFS-08, WFM-01, WFM-02, WFM-03, WFM-04, WFM-05, WFM-06, WFM-08 natural mortality

There is no specific term for lamprey mortality in these areas. Natural mortality across all ages is estimated as

$$M = e^{m} \qquad (3.6.1)$$

where M is natural mortality and m is the log-scale parameter estimate. The initial value for m is specified using Pauly's (1980) equation (Eqn. 1.3), and the deviation from this is included in the likelihood function.

WFH-1-4 and WFH-05 natural mortality

Natural mortality includes lamprey mortality, as in the lake trout models above. It is estimated in the same manner as MI-5, MI-6 and MI-7 (eqn. 2.6.1).

3.7 Standard deviation priors

In all whitefish management areas a variance ratio is used along with a common standard deviation estimated within the model in order to determine the standard deviation priors in the objective function. See Eqn. 2.8.1 above.

WFS-04, WFS-05, WFS-07, WFS-08, WFM-04, WFM-05, WFM-06, WFH-1-4 and WFH-05 standard deviation priors

The ρ in these areas are provided for recruitment, gillnet catch, trap net catch, gillnet effort, trap net effort and the selectivity random walk.

WFM-01, WFM-02 and WFM-08 standard deviation priors

The ρ in these areas are provided for recruitment, trap net catch, trap net effort and the selectivity random walk.

WFM-03 standard deviation priors

The ρ in this area are provided for gillnet catch, trap net catch, gillnet effort, trap net effort and the selectivity random walk.

3.8 Likelihood tables

Below are tables describing the likelihood components for the different areas. D/I distinguishes data-based components (D) from informative priors (I), λ is the weighting factor (if present), L is the form of the likelihood component, and μ, σ and x describe parameters in the normal or lognormal likelihoods. Similar descriptions are not provided for multinomial proportions-at-age because $\hat{p}$ is always the predicted proportion-at-age for the fishery or survey, p is always the observed proportion-at-age and N is always the sample size. All terms in the normal or lognormal log-likelihoods are on a log scale; for example, if the μ column reads "estimated catch" this means estimated catch on a log scale. Some of the likelihood components are coded as normal distributions or as a sum of squares; however if these are equivalent to a different distribution (e.g., lognormal) this is noted.

WFS-04 objective function

Term	D/I	λ	L	μ	σ	x
Commercial TN catch	D	-	$^1n(\mu,\sigma\|x)$	Estimated catch	External rho	Catch data
Commercial GN catch	D	-	$^1n(\mu,\sigma\|x)$	Estimated catch	External rho	Catch data
TN proportions-at-age	D	-	$m(\hat{p}\|p,N)$	-	-	-

Term	D/I	λ	L	μ	σ	x
GN proportions-at-age	D	-	$m(\hat{p}\|p,N)$	-	-	-
TN q RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
GN q RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
TN selectivity k RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
GN selectivity k RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
Natural mortality	I	-	[1]$n(\mu,\sigma\|x)$	Estimate	External fixed	Externally provided (Pauly Eqn.)
Recruitment vector	I	-	$\frac{(\mu-x)^2}{2\sigma^2}$	Ricker recruits	External rho	Age 1 N Estimates

[1]Equivalent to lognormal distribution up to a constant.

WFS-05 objective function

Term	D/I	λ	L	μ	σ	x
Commercial TN catch	D	1	[1]$n(\mu,\sigma\|x)$	Estimated catch	External rho	Catch data
Commercial GN catch	D	1	[1]$n(\mu,\sigma\|x)$	Estimated catch	External rho	Catch data
TN proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-
GN proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-
TN q RW	I	1	$n(\mu,\sigma\|x)$	0	External rho	RW vector
GN q RW	I	1	$n(\mu,\sigma\|x)$	0	External rho	RW vector
TN selectivity k RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
GN selectivity k RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
Natural mortality	I	1	[1]$n(\mu,\sigma\|x)$	Estimate	External fixed	Externally provided (Pauly Eqn.)
Recruitment vector	I	-	$\frac{(\mu-x)^2}{2\sigma^2}$	Ricker recruits	External rho	Age 1 N Estimates

[1]Equivalent to lognormal distribution up to a constant.

WFS-07 objective function

Term	D/I	λ	L	μ	σ	x
Commercial TN catch	D	-	$^{1}n(\mu,\sigma\|x)$	Estimated catch	External rho	Catch data
Commercial GN catch	D	-	$^{1}n(\mu,\sigma\|x)$	Estimated catch	External rho	Catch data
TN proportions-at-age	D	-	$m(\hat{p}\|p,N)$	-	-	-
GN proportions-at-age	D	-	$m(\hat{p}\|p,N)$	-	-	-
TN q RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
GN q RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
TN selectivity k RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
GN selectivity k RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
Natural mortality	I	-	$^{1}n(\mu,\sigma\|x)$	Estimate	External fixed	Externally provided
Recruitment vector	I	-	$n(\mu,\sigma\|x)$	Ricker recruits	External rho	Age 1 N Estimates

[1]Equivalent to lognormal distribution up to a constant.

WFS-08 objective function

Objective function components

Term	D/I	λ	L	μ	σ	x
Commercial TN catch	D	1	$^{1}n(\mu,\sigma\|x)$	Estimated catch	External rho	Catch data
Commercial GN catch	D	1	$^{1}n(\mu,\sigma\|x)$	Estimated catch	External rho	Catch data
TN proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-
GN proportions-at-age	D	1	$m(\hat{p}\|p,N)$	-	-	-

Term	D/I	λ	L	μ	σ	x
TN catchability dev vector	I	1	[1] $n(\mu, \sigma\|x)$	0	External rho	Dev vector
GN catchability dev vector	I	1	[1] $n(\mu, \sigma\|x)$	0	External rho	Dev vector
TN selectivity k RW	I	-	$n(\mu, \sigma\|x)$	0	External rho	RW vector
GN selectivity k RW	I	-	$n(\mu, \sigma\|x)$	0	External rho	RW vector
Natural mortality	I	1	[1] $n(\mu, \sigma\|x)$	Estimate	External fixed	Externally provided
Recruitment vector	I	-	$\frac{(\mu - x)^2}{2\sigma^2}$	Ricker recruits	External rho	Age 1 N Estimates

[1]Equivalent to lognormal distribution up to a constant.

WFM-01 objective function

Objective function components

Term	D/I	λ	L	μ	σ	x
Commercial TN catch	D	-	[1] $n(\mu, \sigma\|x)$	Estimated catch	External rho	Catch data
TN proportions-at-age	D	-	$m(\hat{p}\|p, N)$	-	-	-
TN catchability η RW	I	-	$n(\mu, \sigma\|x)$	0	External rho	RW vector
TN selectivity k RW	I	-	$n(\mu, \sigma\|x)$	0	External rho	RW vector
Natural mortality	I	-	[1] $n(\mu, \sigma\|x)$	Estimate	External fixed	Externally provided
Recruitment vector	I	-	$n(\mu, \sigma\|x)$	Ricker recruits	External rho	Age 1 N Estimates

[1]Equivalent to lognormal distribution up to a constant.

WFM-02 objective function

Objective function components

Term	D/I	λ	L	μ	σ	x
Commercial TN catch	D	-	[1] $n(\mu, \sigma\|x)$	Estimated catch	External rho	Catch data

Term	D/I	λ	L	μ	σ	x
Commercial GN catch	D	-	$^{1}n(\mu,\sigma\|x)$	Estimated catch	External rho	Catch data
TN proportions-at-age	D	-	$m(\hat{p}\|p,N)$	-	-	-
GN proportions-at-age	D	-	$m(\hat{p}\|p,N)$	-	-	-
TN catchability η RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
GN catchability η RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
Natural mortality	I	-	$^{1}n(\mu,\sigma\|x)$	Estimate	External fixed	Externally provided
Recruitment vector	I	-	$n(\mu,\sigma\|x)$	Ricker recruits	External rho	Age 1 N Estimates

[1]Equivalent to lognormal distribution up to a constant.

WFM-03 objective function

Objective function components

Term	D/I	λ	L	μ	σ	x
Commercial TN catch	D	-	$^{1}n(\mu,\sigma\|x)$	Estimated catch	External rho	Catch data
Commercial GN catch	D	-	$^{1}n(\mu,\sigma\|x)$	Estimated catch	External rho	Catch data
TN proportions-at-age	D	-	$m(\hat{p}\|p,N)$	-	-	-
GN proportions-at-age	D	-	$m(\hat{p}\|p,N)$	-	-	-
TN catchability η RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
GN catchability η RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
Natural mortality	I	-	$^{1}n(\mu,\sigma\|x)$	Estimate	External fixed	Externally provided

Term	D/I	λ	L	μ	σ	x
Recruitment vector	I	-	$n(\mu, \sigma \mid x)$	Ricker recruits	External rho	Age 1 N Estimates

[1]Equivalent to lognormal distribution up to a constant.

WFM-04 objective function

Objective function components

Term	D/I	λ	L	μ	σ	x
Commercial TN catch	D	-	[1]$n(\mu, \sigma \mid x)$	Estimated catch	External rho	Catch data
Commercial GN catch	D	-	[1]$n(\mu, \sigma \mid x)$	Estimated catch	External rho	Catch data
TN proportions-at-age	D	-	$m(\hat{p} \mid p, N)$	-	-	-
GN proportions-at-age	D	-	$m(\hat{p} \mid p, N)$	-	-	-
TN catchability η RW	I	-	$n(\mu, \sigma \mid x)$	0	External rho	RW vector
GN catchability η RW	I	-	$n(\mu, \sigma \mid x)$	0	External rho	RW vector
TN selectivity k RW	I	-	$n(\mu, \sigma \mid x)$	0	External rho	RW vector
GN selectivity k RW	I	-	$n(\mu, \sigma \mid x)$	0	External rho	RW vector
Natural mortality	I	-	[1]$n(\mu, \sigma \mid x)$	Estimate	External fixed	Externally provided
Recruitment vector	I	-	$n(\mu, \sigma \mid x)$	Ricker recruits	External rho	Age 1 N Estimates

[1]Equivalent to lognormal distribution up to a constant.

WFM-05 objective function

Objective function components

Term	D/I	λ	L	μ	σ	x
Commercial TN catch	D	-	[1]$n(\mu, \sigma \mid x)$	Estimated catch	External rho	Catch data
Commercial GN	D	-	[1]$n(\mu, \sigma \mid x)$	Estimated catch	External rho	Catch data

Term	D/I	λ	L	μ	σ	x
catch						
TN proportions-at-age	D	-	$m(\hat{p}\|p,N)$	-	-	-
GN proportions-at-age	D	-	$m(\hat{p}\|p,N)$	-	-	-
TN catchability η RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
GN catchability η RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
TN selectivity k RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
GN selectivity kRW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
Natural mortality	I	-	[1]$n(\mu,\sigma\|x)$	Estimate	External fixed	Externally provided
Recruitment vector	I	-	$n(\mu,\sigma\|x)$	Ricker recruits	External rho	Age 1 N Estimates

[1]Equivalent to lognormal distribution up to a constant.

WFM-06 objective function

Objective function components

Term	D/I	λ	L	μ	σ	x
Commercial TN catch	D	-	[1]$n(\mu,\sigma\|x)$	Estimated catch	External rho	Catch data
Commercial GN catch	D	-	[1]$n(\mu,\sigma\|x)$	Estimated catch	External rho	Catch data
TN proportions-at-age	D	-	$m(\hat{p}\|p,N)$	-	-	-
GN proportions-at-age	D	-	$m(\hat{p}\|p,N)$	-	-	-
TN catchability η RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
GN catchability η RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
TN selectivity k RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector

Term	D/I	λ	L	μ	σ	x
GN selectivity k RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
Natural mortality	I	-	[1]$n(\mu,\sigma\|x)$	Estimate	External fixed	Externally provided
Recruitment vector	I	-	$n(\mu,\sigma\|x)$	Ricker recruits	External rho	Age 1 N Estimates

[1]Equivalent to lognormal distribution up to a constant.

WFM-08 objective function

Objective function components

Term	D/I	λ	L	μ	σ	x
Commercial TN catch	D	-	[1]$n(\mu,\sigma\|x)$	Estimated catch	External rho	Catch data
TN proportions-at-age	D	-	$m(\hat{p}\|p,N)$	-	-	-
TN catchability η RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
TN selectivity k RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
Natural mortality	I	-	[1]$n(\mu,\sigma\|x)$	Estimate	External fixed	Externally provided
Recruitment vector	I	-	$n(\mu,\sigma\|x)$	Ricker recruits	External rho	Age 1 N Estimates

[1]Equivalent to lognormal distribution up to a constant.

H-1-4 objective function

Objective function components

Term	D/I	λ	L	μ	σ	x
Commercial TN catch	D	-	[1]$n(\mu,\sigma\|x)$	Estimated catch	External rho	Catch data
Commercial GN catch	D	-	[1]$n(\mu,\sigma\|x)$	Estimated catch	External rho	Catch data
TN proportions-at-age	D	-	$m(\hat{p}\|p,N)$	-	-	-
GN proportions-at-	D	-	$m(\hat{p}\|p,N)$	-	-	-

Term	D/I	λ	L	μ	σ	x
age						
TN catchability η RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
GN catchability η RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
TN selectivity k RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
GN selectivity k RW	I	-	$n(\mu,\sigma\|x)$	0	External rho	RW vector
Natural mortality	I	-	[1]$n(\mu,\sigma\|x)$	Estimate	External fixed	Externally provided
Recruitment vector	I	-	$n(\mu,\sigma\|x)$	Ricker recruits	External rho	Age 1 N Estimates

[1]Equivalent to lognormal distribution up to a constant.

WFH-05 objective function

Objective function components

Term	D/I	λ	L	μ	σ	x
[2]Commercial TN catch	D	-	[1]$n(\mu,\sigma\|x)$	Estimated catch	External fixed	Catch data
TN proportions-at-age	D	-	$m(\hat{p}\|p,N)$	-	-	-
TN q_{1981}	I	-	$n(\mu,\sigma\|x)$	-1.9	0.10	$\bar{q}$
TN catchability η RW	I	-	$n(\mu,\sigma\|x)$	0	External fixed	RW vector
TN selectivity vector λ	I	-	$n(\mu,\sigma\|x)$	0	External rho	vector
TN selectivity ϕ	I	-	$n(\mu,\sigma\|x)$	2.35	0.08	σ
TN selectivity μ_b	I	-	$n(\mu,\sigma\|x)$	1.84	0.02	μ
Natural mortality	I	-	$n(\mu,\sigma\|x)$	Estimate	External fixed	Externally provided
Recruitment vector	I	-	$n(\mu,\sigma\|x)$	Ricker recruits	External fixed	Age 1 N Estimates
Recruitment vector	I	-	[1]$n(\mu,\sigma\|x)$	0	External fixed	vector

[1]Equivalent to lognormal distribution up to a constant.
[2]The gillnet fishery, while available, is not included in the likelihood.

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Appendix

Below are listed summary tables for comparing the lake trout assessment areas. Each cell in the tables is described in more detail in the text.

Table A1: Selectivity. "L" stands for logistic, "DL" stands for double logistic, "LN" for lognormal, "RW" for random walk, "Blocks" indicates independent estimates for different time blocks and "quad" stands for quadratic equation, "Ref" stands for reference length and "Dv" stands for dev vector.

Area	Function	Length / Age	Standardization	Time-varying (com)	Time-varying (rec)	Time-varying (survey1)	Time-varying (survey2)
MI-5	Gamma	A	Max	RW	No	Random walk	No
MI-6	Gamma	A	Max	RW	RW	RW	No
MI-7	Gamma	A	Max	RW	RW	RW	No
MM-123	DL	A	Max	No	Blocks	No	-
MM-4	DL	A	Max	No	Blocks	RW	-
MM-5	DL	A	Max	No	Blocks	RW	-
MM-67	DL	A	Max	Quad	Quad	Quad	-
MH-1	LN	?	Analytical	Blocks	Blocks	-	-
MH-2	LN	?	Analytical	Blocks	Blocks	-	-
WFS-04	L/DL	L	Ref	RW	-	-	-
WFS-05	Gamma	L	Max	RW	-	-	-
WFS-07	DL	L	Ref	RW	-	-	-
WFS-08	DL	A	Ref	RW	-	-	-
WFM-01	DL	L	Ref	RW	-	-	-
WFM-02	L/DL	L	Max	Quad	-	-	-
WFM-03	L/DL	L	Max	Quad	-	-	-
WFM-04	L/DL	L	Ref	RW	-	-	-
WFM-05	L/DL	L	Ref	RW	-	-	-
WFM-06	L/DL	L	Ref	RW	-	-	-
WFM-08	L	L	Ref	RW	-	-	-

Area	Function	Length / Age	Standardization	Time-varying (com)	Time-varying (rec)	Time-varying (survey1)	Time-varying (survey2)
WFH-1-4	DL	L	Ref	RW	-	-	-
WFH-05	LN	L	Analytical	RW/Dv			

Table A2: Catchability. "Dv" stands for dev vector (a vector that sums to zero) and "RW" for random walk.

Area	Time-varying (comm)	Time-varying (rec)	Time-varying (survey1)	Time-varying (survey2)
MI-5	Dv	Dv	No	No
MI-6	Dv	Dv	No	No
MI-7	Dv	Dv	No	No
MM-123	RW	RW	No	-
MM-4	RW	RW	No	-
MM-5	RW	RW	No	-
MM-67	NA	Dv	No	-
MH-1	Independent	RW	-	-
MH-2	Independent	RW	-	-
WFS-04	RW	-	-	-
WFS-05	RW	-	-	-
WFS-07	RW	-	-	-
WFS-08	Dv	-	-	-
WFM-01	RW	-	-	-
WFM-02	RW	-	-	-
WFM-03	RW	-	-	-
WFM-04	RW	-	-	-
WFM-05	RW	-	-	-
WFM-06	RW	-	-	-

Area	Time-varying (comm)	Time-varying (rec)	Time-varying (survey1)	Time-varying (survey2)
WFM-08	RW	-	-	-
WFH-1-4	RW	-	-	-
WFH-05	RW	-	-	-

Table A3: Recruitment. "Dv" stands for dev vector, "UDv" for unconstrained dev vector and "RW" for random walk.

Area	Wild rec	Stocked Recruitment
MI-5	Dv	NA
MI-6	RW	NA
MI-7	RW	NA
MM-123	NA	Movement matrix and fingerling conversion
MM-4	NA	Movement matrix and fingerling conversion
MM-5	NA	Movement matrix and fingerling conversion
MM-67	NA	Movement matrix and fingerling conversion
MH-1	Independent estimate	Movement matrix and fingerling conversion
MH-2	Independent estimate	Movement matrix and fingerling conversion
WFS-04	UDv[1]	NA
WFS-05	UDv[1]	NA
WFS-07	UDv[1]	NA
WFS-08	UDv[1]	NA
WFM-01	UDv[1]	NA
WFM-02	UDv[1]	NA
WFM-03	UDv[1]	NA
WFM-04	UDv[1]	NA
WFM-05	UDv[1]	NA

Area	Wild rec	Stocked Recruitment
WFM-06	UDv[1]	NA
WFM-08	UDv[1]	NA
WFH-1-4	UDv[1]	NA
WFH-05	UDv[1]	NA

[1]While the vectors of deviations are not constrained to sum to zero as a vector of recruits, the recruitment deviations are included in the same vector as the initial population estimates, and these deviations together are constrained to sum to zero. See text for details.

Table A4: Numbers-at-age and catch-at-age.

Area	Initial year	Plus group	Ageing error
MI-5	Ages 2-7 independent estimates; 7+ exponential survival	Simple exponential survival	Ageing error matrix
MI-6	Independent estimates	Simple exponential survival	Ageing error matrix
MI-7	Independent estimates	Simple exponential survival	Ageing error matrix
MM-123	Independent estimates	Simple exponential survival	Assumed error-fee
MM-4	Independent estimates	Simple exponential survival	Assumed error-fee
MM-5	Independent estimates	Simple exponential survival	Assumed error-fee
MM-67	Independent estimates	Simple exponential survival	Assumed error-fee
MH-1	Independent estimates	Infinite series	Assumed error-fee
MH-2	Independent estimates	Infinite series	Assumed error-fee
WFS-04	UDv[1]	Simple exponential survival	Assumed error-fee
WFS-05	UDv[1]	Simple exponential survival	Assumed error-fee
WFS-07	UDv[1]	Simple exponential survival	Assumed error-fee
WFS-08	Udv[1]	Simple exponential survival	Assumed error-fee
WFM-01	Udv[1]	Simple exponential survival	Assumed error-fee
WFM-02	Udv[1]	Simple exponential survival	Assumed error-fee
WFM-03	Udv[1]	Simple exponential survival	Assumed error-fee
WFM-04	Udv[1]	Simple exponential survival	Assumed error-fee

Area	Initial year	Plus group	Ageing error
WFM-05	Udv[1]	Simple exponential survival	Assumed error-fee
WFM-06	Udv[1]	Simple exponential survival	Assumed error-fee
WFM-08	Udv[1]	Simple exponential survival	Assumed error-fee
WFH-1-4	Udv[1]	Simple exponential survival	Assumed error-fee
WFH-05	Udv	Simple exponential survival	Assumed error-fee

[1]While the vectors of deviations are not constrained to sum to zero as a vector of initial population size, the initial population deviations are included in the same vector as the recruitment estimates, and these deviations together are constrained to sum to zero. See text for details.

Table A5: Natural mortality.

Area	Age-1 mortality	Background M method	Lamprey Mortality
MI-5	NA	Independent estimate	External
MI-6	NA	Independent estimate	External
MI-7	NA	Independent estimate	External
MM-123	Independent annual estimate	Independent estimate	External
MM-4	Independent annual estimate	Independent estimate	External
MM-5	Independent annual estimate	Independent estimate	External
MM-67	Independent annual estimate	Independent estimate	External
MH-1	Independent annual estimate	Length-based	External
MH-2	Independent annual estimate	Length-based	External
WFS-04	NA	Independent estimate	-
WFS-05	NA	Independent estimate	-
WFS-07	NA	Independent estimate	-
WFS-08	NA	Independent estimate	-

Area	Age-1 mortality	Background M method	Lamprey Mortality
WFM-01	NA	Independent estimate	-
WFM-02	NA	Independent estimate	-
WFM-03	NA	Independent estimate	-
WFM-04	NA	Independent estimate	-
WFM-05	NA	Independent estimate	-
WFM-06	NA	Independent estimate	-
WFM-08	NA	Independent estimate	-
WFH-1-4	NA	Independent estimate	External
WFH-05	NA	Independent estimate	External

Table A6: Standard deviation priors. If some variables use an estimated common σ with externally provided ρ, the model is labeled as such in the table. This does not mean that every single σ is treated this way in those model however; see text for details.

Area	Prior method
MI-5	Externally provided σ
MI-6	Externally provided σ
MI-7	Externally provided σ
MM-123	Estimated common σ with externally provided ρ
MM-4	Estimated common σ with externally provided ρ
MM-5	Estimated common σ with externally provided ρ
MM-67	Externally provided σ
MH-1	Externally provided common σ and externally provided ρ
MH-2	Externally provided common σ and externally provided ρ
WFS-04	Estimated common σ with externally provided ρ
WFS-05	Estimated common σ with externally provided ρ
WFS-07	Estimated common σ with externally provided ρ
WFS-08	Estimated common σ with externally provided ρ

Area	Prior method
WFM-01	Estimated common σ with externally provided ρ
WFM-02	Estimated common σ with externally provided ρ
WFM-03	Estimated common σ with externally provided ρ
WFM-04	Estimated common σ with externally provided ρ
WFM-05	Estimated common σ with externally provided ρ
WFM-06	Estimated common σ with externally provided ρ
WFM-08	Estimated common σ with externally provided ρ
WFH-1-4	Estimated common σ with externally provided ρ
WFH-05	Estimated common σ with externally provided ρ

Table A7: Objective function log likelihoods. See section 2.10.

Area	Catch	Survey CPUE	Com q	Rec q	Survey q
MI-5	[1]$n(\mu,\sigma\|x)$	[1]$n(\mu,\sigma\|x)$	[1]$n(\mu,\sigma\|x)$	[1]$n(\mu,\sigma\|x)$	-
MI-6	[1]$n(\mu,\sigma\|x)$	[1]$n(\mu,\sigma\|x)$	[1]$n(\mu,\sigma\|x)$	[1]$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$
MI-7	[1]$n(\mu,\sigma\|x)$	[1]$n(\mu,\sigma\|x)$	[1]$n(\mu,\sigma\|x)$	[1]$n(\mu,\sigma\|x)$	-
MM-123	[1]$n(\mu,\sigma\|x)$	[1]$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	-
MM-4	[1]$n(\mu,\sigma\|x)$	[1]$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	-
MM-5	[1]$n(\mu,\sigma\|x)$	[1]$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	-
MM-67	[1]$n(\mu,\sigma\|x)$	[1]$n(\mu,\sigma\|x)$	-	[1]$n(\mu,\sigma\|x)$	-
MH-1	[1]$n(\mu,\sigma\|x)$	[1]$n(\mu,\sigma\|x)$	$l(\mu,\sigma\|x)$	$l(\mu,\sigma\|x)$	$l(\mu,\sigma\|x)$
MH-2	[1]$n(\mu,\sigma\|x)$	[1]$n(\mu,\sigma\|x)$	$l(\mu,\sigma\|x)$	$l(\mu,\sigma\|x)$	$l(\mu,\sigma\|x)$
WFS-04	[1]$n(\mu,\sigma\|x)$	-	$n(\mu,\sigma\|x)$	-	-
WFS-05	[1]$n(\mu,\sigma\|x)$	-	$n(\mu,\sigma\|x)$	-	-
WFS-07	[1]$n(\mu,\sigma\|x)$	-	$n(\mu,\sigma\|x)$	-	-
WFS-08	[1]$n(\mu,\sigma\|x)$	-	[1]$n(\mu,\sigma\|x)$	-	-
WFM-01	[1]$n(\mu,\sigma\|x)$	-	$n(\mu,\sigma\|x)$	-	-

Area	Catch	Survey CPUE	Com q	Rec q	Survey q
WFM-02	[1]$n(\mu,\sigma\|x)$	-	$n(\mu,\sigma\|x)$	-	-
WFM-03	[1]$n(\mu,\sigma\|x)$	-	$n(\mu,\sigma\|x)$	-	-
WFM-04	[1]$n(\mu,\sigma\|x)$	-	$n(\mu,\sigma\|x)$	-	-
WFM-05	[1]$n(\mu,\sigma\|x)$	-	$n(\mu,\sigma\|x)$	-	-
WFM-06	[1]$n(\mu,\sigma\|x)$	-	$n(\mu,\sigma\|x)$	-	-
WFM-08	[1]$n(\mu,\sigma\|x)$	-	$n(\mu,\sigma\|x)$	-	-
WFH-1-4	[1]$n(\mu,\sigma\|x)$	-	$n(\mu,\sigma\|x)$	-	-
WFH-05	[1]$n(\mu,\sigma\|x)$	-	$n(\mu,\sigma\|x)$	-	-

Table A7 continued

Area	Prop.-at-age	Background M	Recruitment	Selectivity par	Age-1 M
MI-5	$m(\hat{p}\|p,N)$	[1]$n(\mu,\sigma\|x)$	[1]$\sum x^2$	$n(\mu,\sigma\|x)$	-
MI-6	$m(\hat{p}\|p,N)$	[1]$n(\mu,\sigma\|x)$	$\frac{\sum x^2}{2\sigma^2}$	$n(\mu,\sigma\|x)$	-
MI-7	$m(\hat{p}\|p,N)$	[1]$n(\mu,\sigma\|x)$	$\frac{\sum x^2}{2\sigma^2}$	$n(\mu,\sigma\|x)$	-
MM-123	$m(\hat{p}\|p,N)$	-	-	-	$n(\mu,\sigma\|x)$
MM-4	$m(\hat{p}\|p,N)$	-	-	$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$
MM-5	$m(\hat{p}\|p,N)$	-	-	$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$
MM-67	$m(\hat{p}\|p,N)$	-	-	-	$n(\mu,\sigma\|x)$
MH-1	$m(\hat{p}\|p,N)$	$l(\mu,\sigma\|x)$	-	$l(\mu,\sigma\|x)$	$l(\mu,\sigma\|x)$
MH-2	$m(\hat{p}\|p,N)$	$l(\mu,\sigma\|x)$	-	$l(\mu,\sigma\|x)$	$l(\mu,\sigma\|x)$
WFS-04	$m(\hat{p}\|p,N)$	[1]$n(\mu,\sigma\|x)$	$\frac{(\mu-x)^2}{2\sigma^2}$	$n(\mu,\sigma\|x)$	-
WFS-05	$m(\hat{p}\|p,N)$	[1]$n(\mu,\sigma\|x)$	$\frac{(\mu-x)^2}{2\sigma^2}$	$n(\mu,\sigma\|x)$	-
WFS-07	$m(\hat{p}\|p,N)$	[1]$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	-
WFS-08	$m(\hat{p}\|p,N)$	[1]$n(\mu,\sigma\|x)$	$\frac{(\mu-x)^2}{2\sigma^2}$	$n(\mu,\sigma\|x)$	-
WFM-01	$m(\hat{p}\|p,N)$	[1]$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	-

Area	Prop.-at-age	Background M	Recruitment	Selectivity par	Age-1 M
WFM-02	$m(\hat{p}\|p,N)$	[1]$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	-	-
WFM-03	$m(\hat{p}\|p,N)$	[1]$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	-	-
WFM-04	$m(\hat{p}\|p,N)$	[1]$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	-
WFM-05	$m(\hat{p}\|p,N)$	[1]$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	-
WFM-06	$m(\hat{p}\|p,N)$	[1]$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	-
WFM-08	$m(\hat{p}\|p,N)$	[1]$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	-
WFH-1-4	$m(\hat{p}\|p,N)$	[1]$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	-
WFH-05	$m(\hat{p}\|p,N)$	[1]$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	$n(\mu,\sigma\|x)$	-

[1]Equivalent to lognormal distribution up to a constant.

Table A8 Additional model summaries.

Area	Description	Model implementation
	Model range	1975-2013
	Age range	4-15
	Large mesh selectivity random walk	1976-2011
MI-5	Commercial fishery selectivity random walk	1987-1999
	Commercial fishery selectivity time blocks	2000, 2003-2004, 2006-2013, 2001-2002, 2005
	Commercial fishery catchability dev vector	1986-2013
	Recreational fishery catchability dev vector	1984-1988, 1990-2013
	Model range	1978-2013
	Age range	4-15
	Recreational fishery selectivity random walk	1988-2005, 2007-2011
MI-6	Large mesh survey selectivity random walk	1979-2011
	Commercial fishery selectivity random walk	1979-2011
	Recreational fishery catchability random walk	1991-2013
	Commercial fishery catchability random walk	1978-2013

MI-7	Model range	1975-2011
	Age range	4-15
	Recreational fishery selectivity random walk	1985-2009
	Large mesh fishery selectivity random walk	1976-2009
	Commercial fishery selectivity random walk	1976-2009
	Commercial catchability dev vector	1980-2011
	Recreational fishery catchability dev vector	1984-1988, 1990-2011
MM-123	Model range	1981-2013
	Age range	1-15
	Recreational fishery selectivity time blocks	[1985-1991], [1992-2011], 2012-2013
	Commercial fishery selectivity random walk	1982-2013
	Recreational fishery catchability random walk	1986-2013
	Commercial fishery catchability random walk	1982-2013
MM-4	Model range	1981-2013
	Age range	1-15
	Recreational fishery selectivity time blocks	1985-1991, 1992-1996, 1997-2002, 2003-2005, 2006-2010, 2011-2013
	Survey selectivity random walk	1982-2013
	Commercial fishery catchability random walk	1982-2013
	Recreational fishery catchability random walk	1986-2013
MM-5	Model range	1981-2013
	Age range	1-15
	Recreational fishery selectivity time blocks	1985-2000, 2001-2002, 2003-2005, 2006-2011, 2012-2013
	Survey selectivity random walk	1982-1989, 1997-2010
	Commercial fishery catchability random walk	1982-1989, 1996-2007

	Recreational fishery catchability random walk	1982-2013
MM-67	Model range	1981-2011
	Model ages	1-15
	Recreational fishery selectivity time blocks	1985-2002, 2003-2011
	Commercial fishery selectivity quadratic	1982-2011
	Survey selectivity quadratic	1982-2011
	Recreational fishery catchability dev vector	1985-2011
MH-1	Model range	1977-2013
	Model ages	1-15
	Recreational fishery catchability random walk	1986-2013
MH-2	Model range	1984-2013
	Model ages	1-15
	Recreational fishery catchability random walk	1986-2013
WFS-04	Model range	1986-2012
	Age range	4-12
	Gill net and trap net selectivity random walk years	1987-2012
	Gill net and trap net catchability random walk years	1987-2012
WFS-05	Model range	1986-2012
	Age range	4-12
	Gill net and trap net selectivity random walk years	1987-2012
	Gill net and trap net catchability random walk years	1987-2012
WFS-07	Model range	1976-2012
	Age range	4-11
	Gill net and trap net selectivity random walk years	1977-2012

	Gill net and trap net catchability random walk years	1977-2012
WFS-08	Model range	1981-2012
	Age range	4-11
	Gill net selectivity random walk	1982-2012
	Trap net selectivity random walk	1997-2012
WFM-01	Model range	1976-2012
	Age range	3-12
	Trap net selectivity random walk	1977-2012
	Trap net catchability random walk	1977-2012
WFM-02	Model range	1986-2012
	Age range	3-12
	Gill net quadratic selectivity	1986-1989, 1993-2012
	Trap net quadratic selectivity	1986-1987, 1991–2003, 2012
	Gill net and trap net catchability random walk	1987-2012
WFM-03	Model range	1986-2012
	Age range	4-15
	Gill net and trap net quadratic selectivity	1986-2012
	Gill net and trap net catchability random walk	1987-2012
WFM-04	Model range	1981-2012
	Age range	3-11
	Gill net random walk selectivity	1987-2012
	Trap net random walk selectivity	1990-2012
	Gill net random walk catchability	1982-2012
	Trap net random walk catchability	1982-1986, 1989-2012
WFM-05	Model range	1981-2012
	Age range	3-12

	Gill net and trap net random walk selectivity	1982-2012
	Gill net and trap net random walk catchability	1982-2012
WFM-06	Model range	1985-2012
	Age range	3-12
	Gill net random walk selectivity	1994-2012
	Trap net random walk selectivity	1986-2012
	Gill net random walk catchability	1986-1989, 1993-2012
	Trap net random walk catchability	1986-2012
WFM-08	Model range	1985-2012
	Age range	3-12
	Trap net random walk selectivity	1986-2012
	Trap net random walk catchability	1986-2012
WFH-1-4	Model range	1976-2012
	Age range	4-12
	Gill net random walk selectivity	1983-2012
	Trap net random walk selectivity	1977-2012
	Gill net and trap net random walk catchability	1977-2012
WFH-05	Model range	1981-2012
	Age range	3-15
	Gill net random walk selectivity	1984-2012
	Trap net annual variability	1981-2012
	Gill net and trap net random walk catchability	1982-2012